Euler's Theorems and Fleury's Algorithm

Lecture 27
Section 5.3

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Mon, Nov 5, 2018

1. Definitions
2. Euler's Theorems
3. Fleury's Algorithm
4. The Splicing Algorithm
5. Example
6. The Mail Carrier Problem Solved
7. Assignment

Outline

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Euler Paths and Circuits

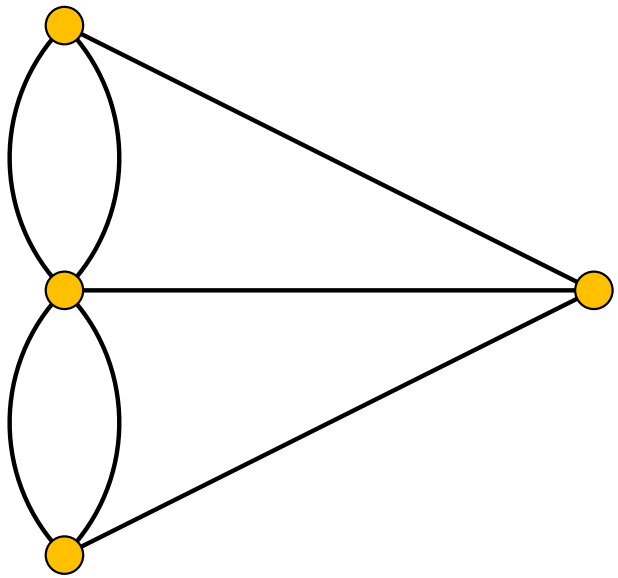
Definition (Euler Path)

An Euler path (pronounced "oiler") is a path that traverses each edge exactly once.

Definition (Euler Circuit)

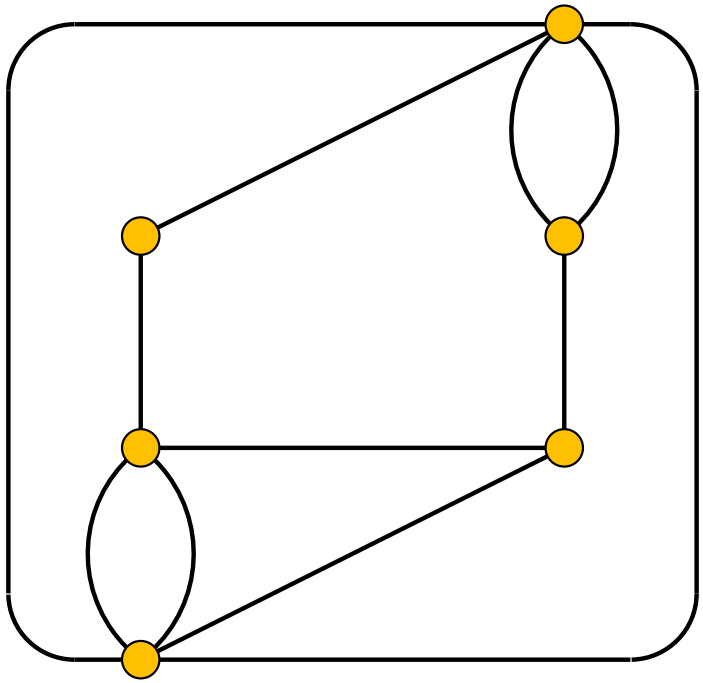
An Euler circuit is an Euler path that is a circuit.

Euler Paths and Circuits



- In the Seven Bridges of Königsberg, we would like to find an Euler circuit.

Euler Paths and Circuits



- In the Bridges of Madison County, we would like to find an Euler circuit, but, failing that, we want to minimize the number of repeated edges (different problem).

Definitions

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A graph is connected if, for any two vertices, there is a path from one to the other.

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The degree of a vertex is the number of edges that emanate from it (that are “incident” to it).

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Definition (Even and Odd Vertices)

A vertex is even if it has even degree. Otherwise, it is odd.

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Euler's Theorems

Theorem (Euler Circuits)

If a graph

- *is connected and*
- *every vertex is even,*

then

- *it has an Euler circuit.*

Otherwise, it does not have an Euler circuit.

Euler's Theorems

Theorem (Euler Paths)

If a graph

- *is connected and*
- *has exactly 2 odd vertices,*

then

- *it has an Euler path and*
- *any Euler path must begin at one of the odd vertices and end that the other one.*

Euler's Theorems

Theorem (Euler Paths)

If a graph

- *is not connected or*
- *has more than 2 odd vertices,*

then it does not have an Euler path.

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- Whenever there is a choice of edges, choose an edge that does not separate you from any remaining (unvisited) edges. (This requires thought.) (Not much, but some.)
- Continue to follow edges in this manner.
- When you reach the starting point, you have an Euler circuit.

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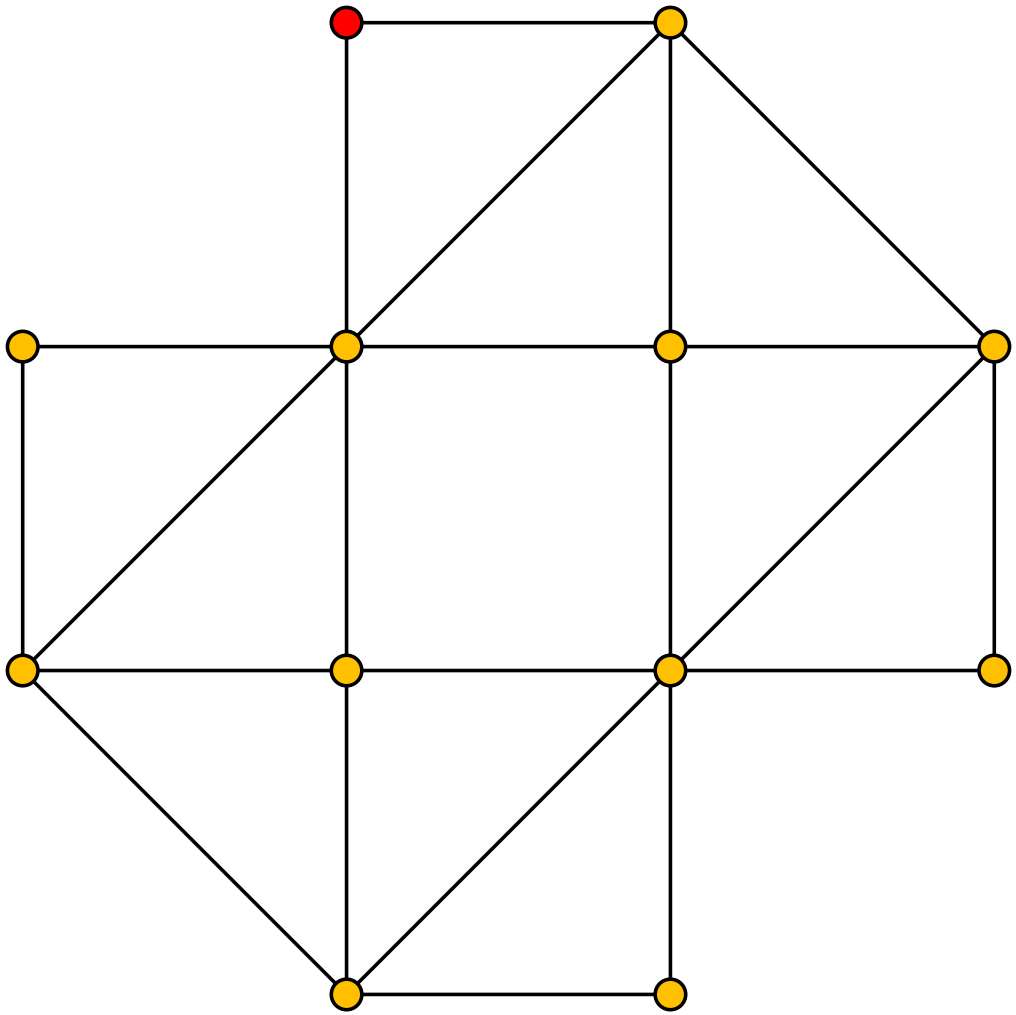

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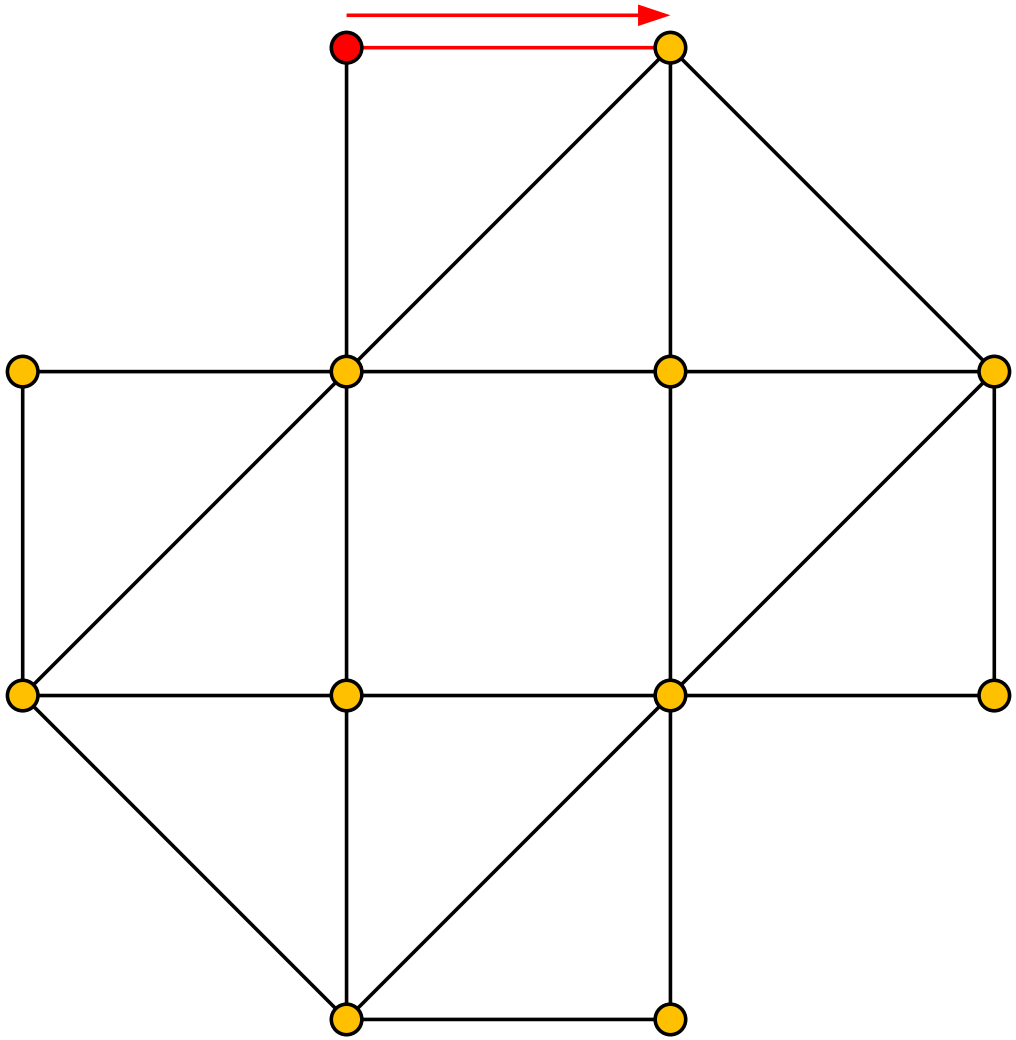

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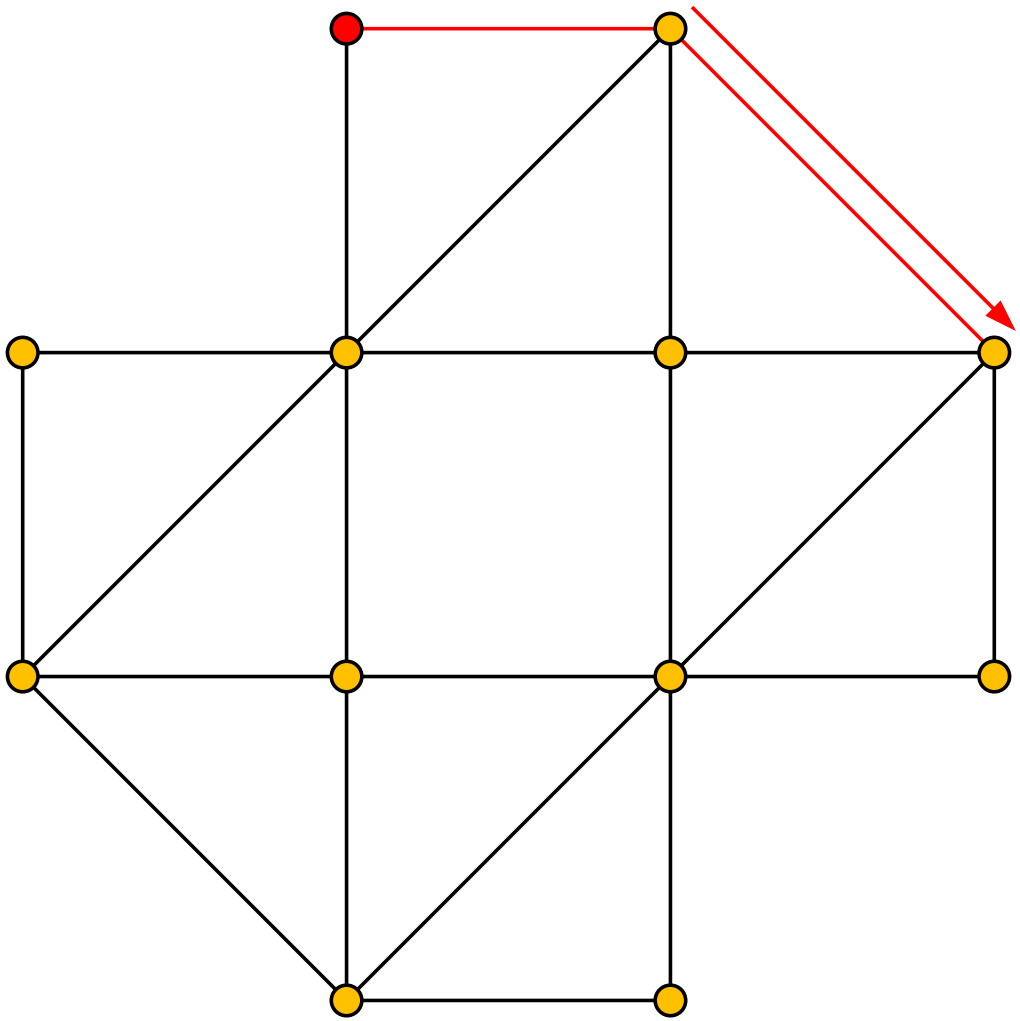

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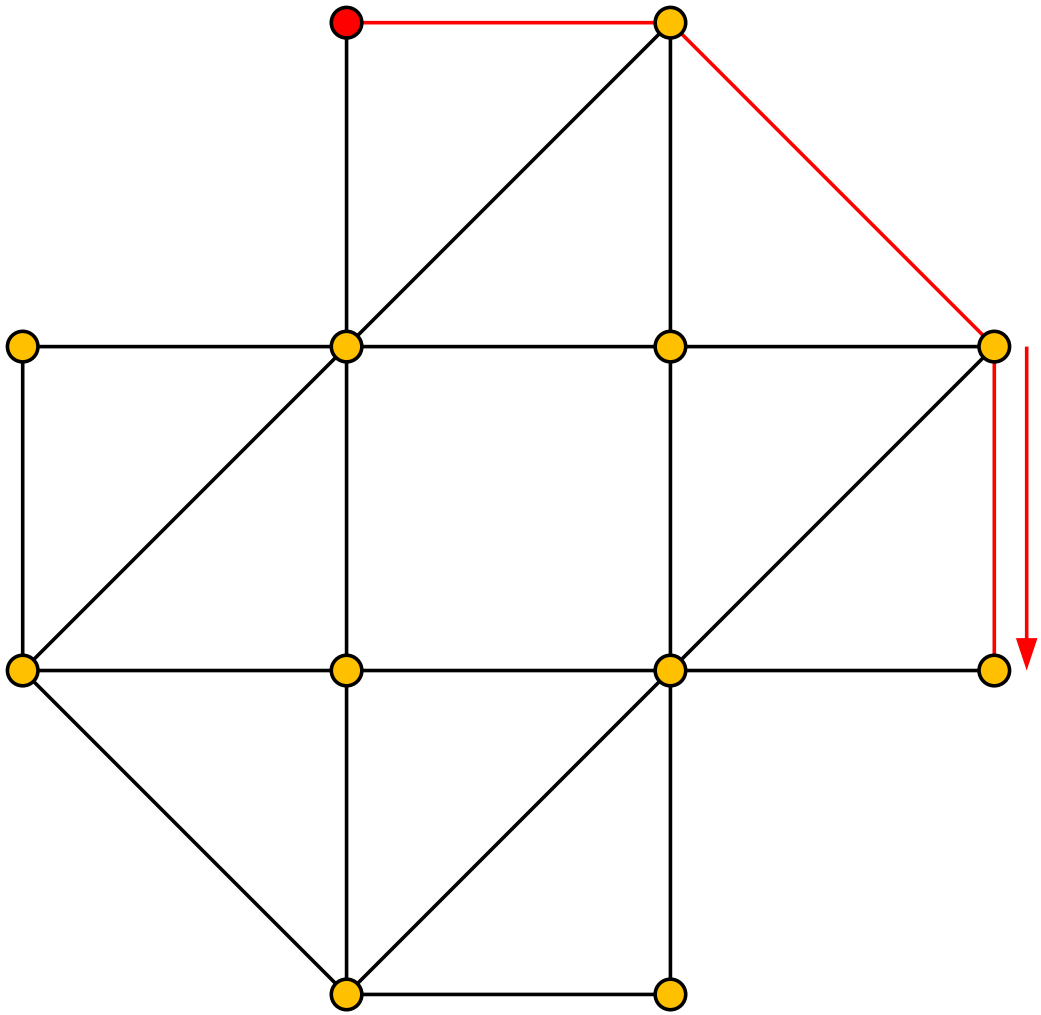

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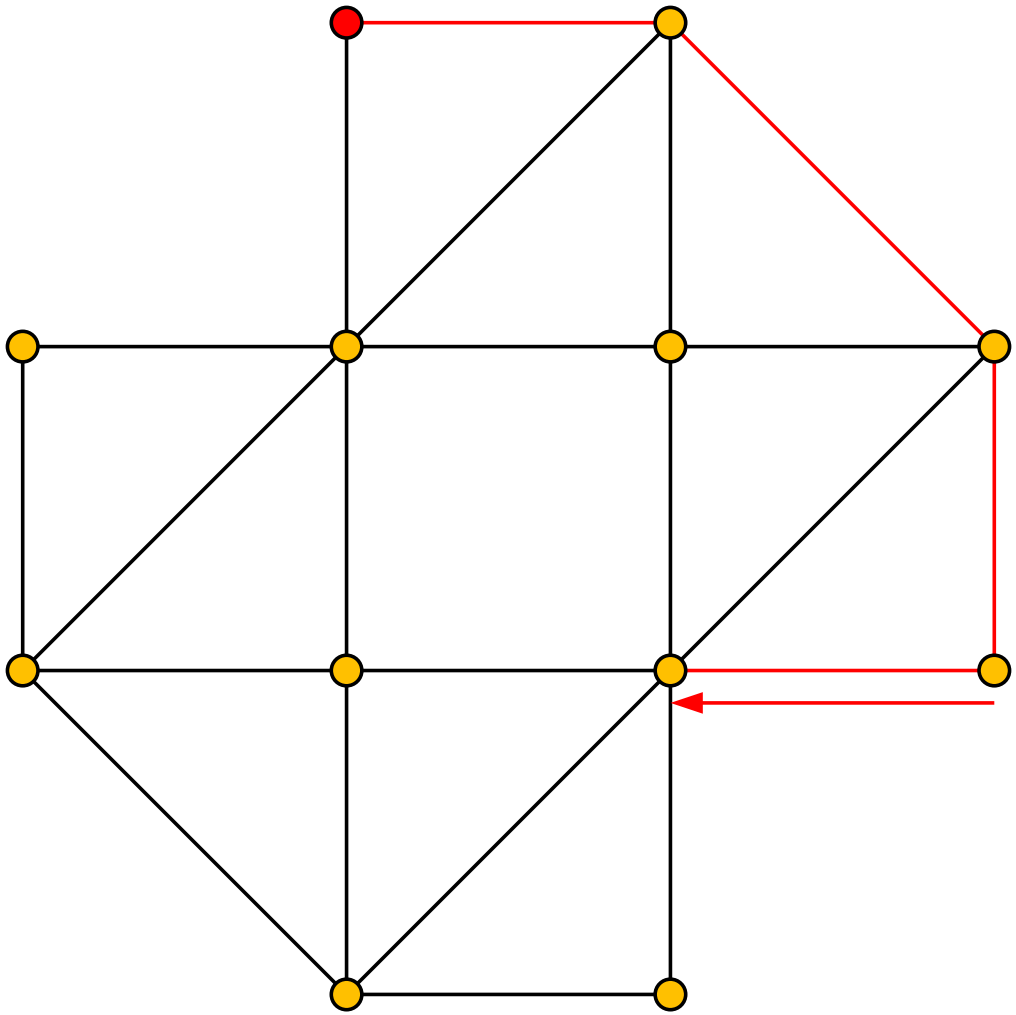

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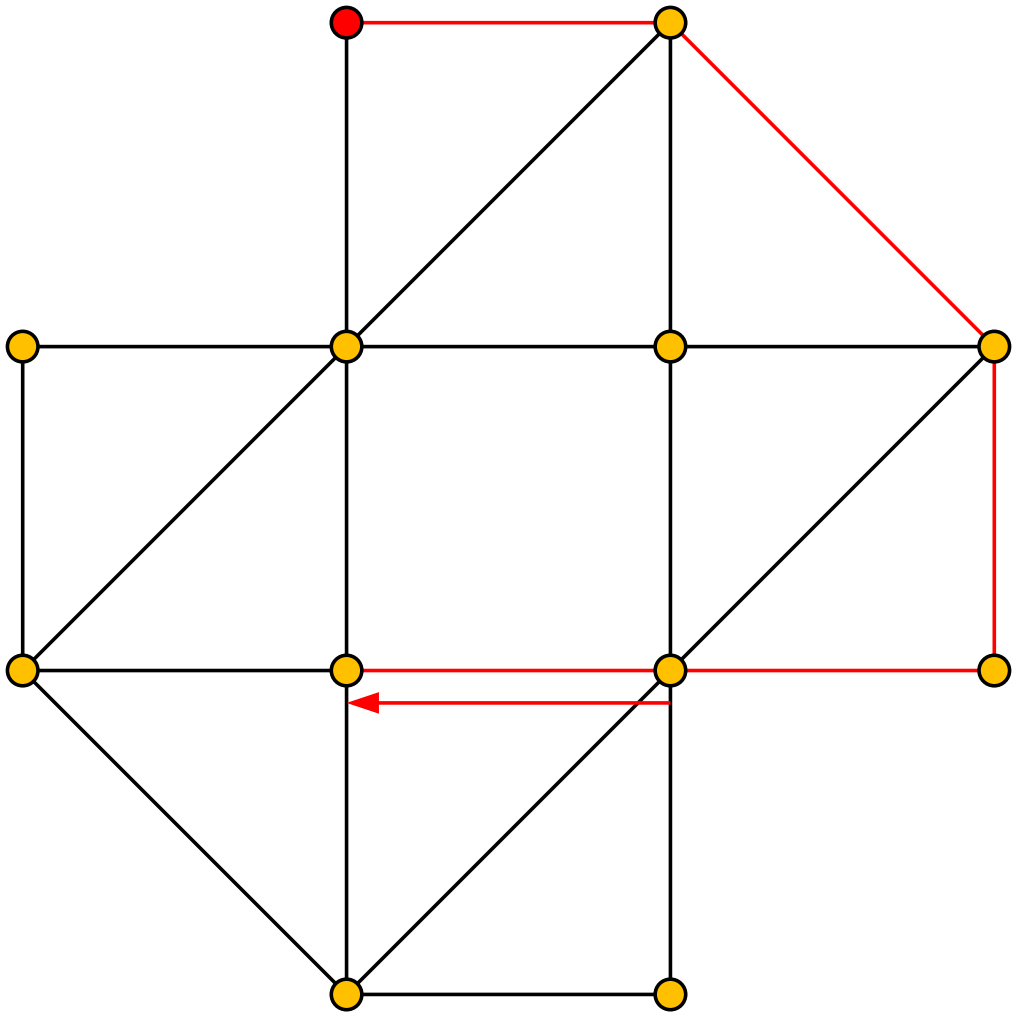

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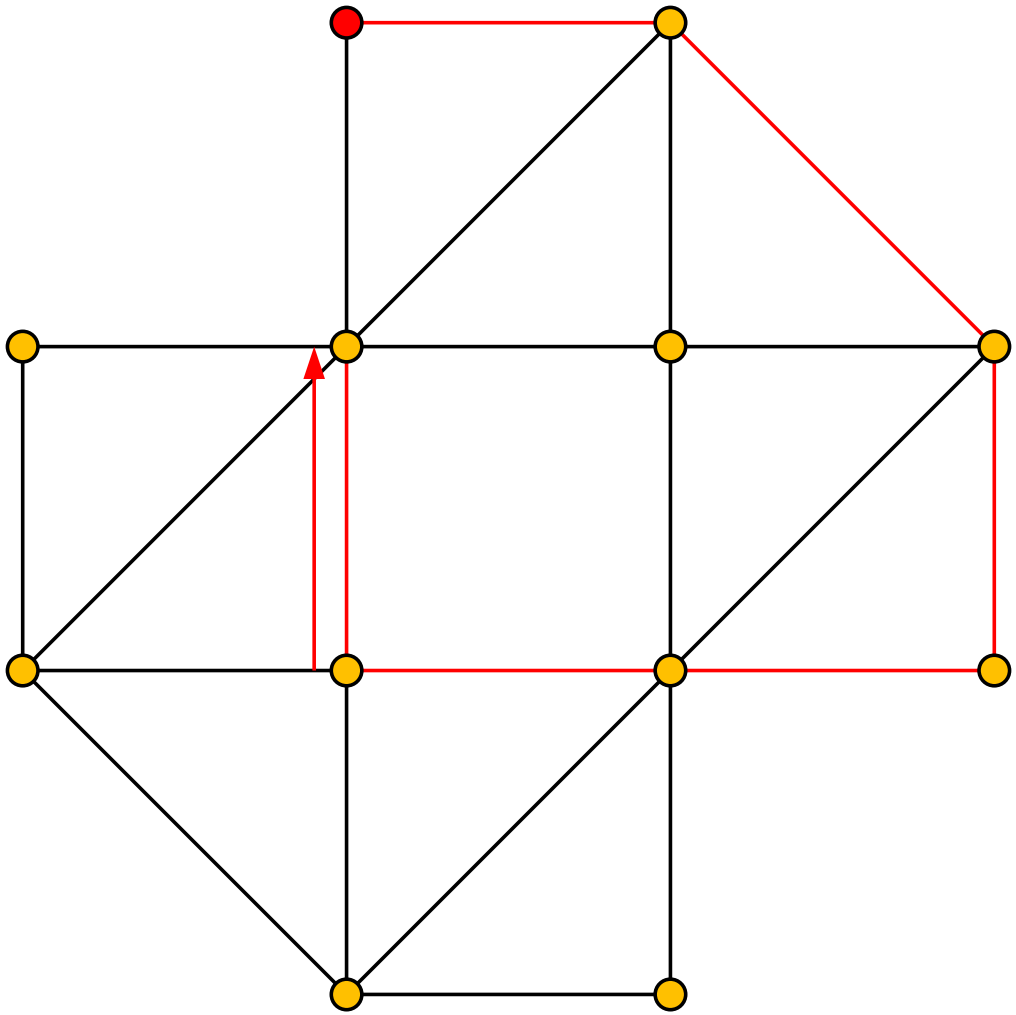

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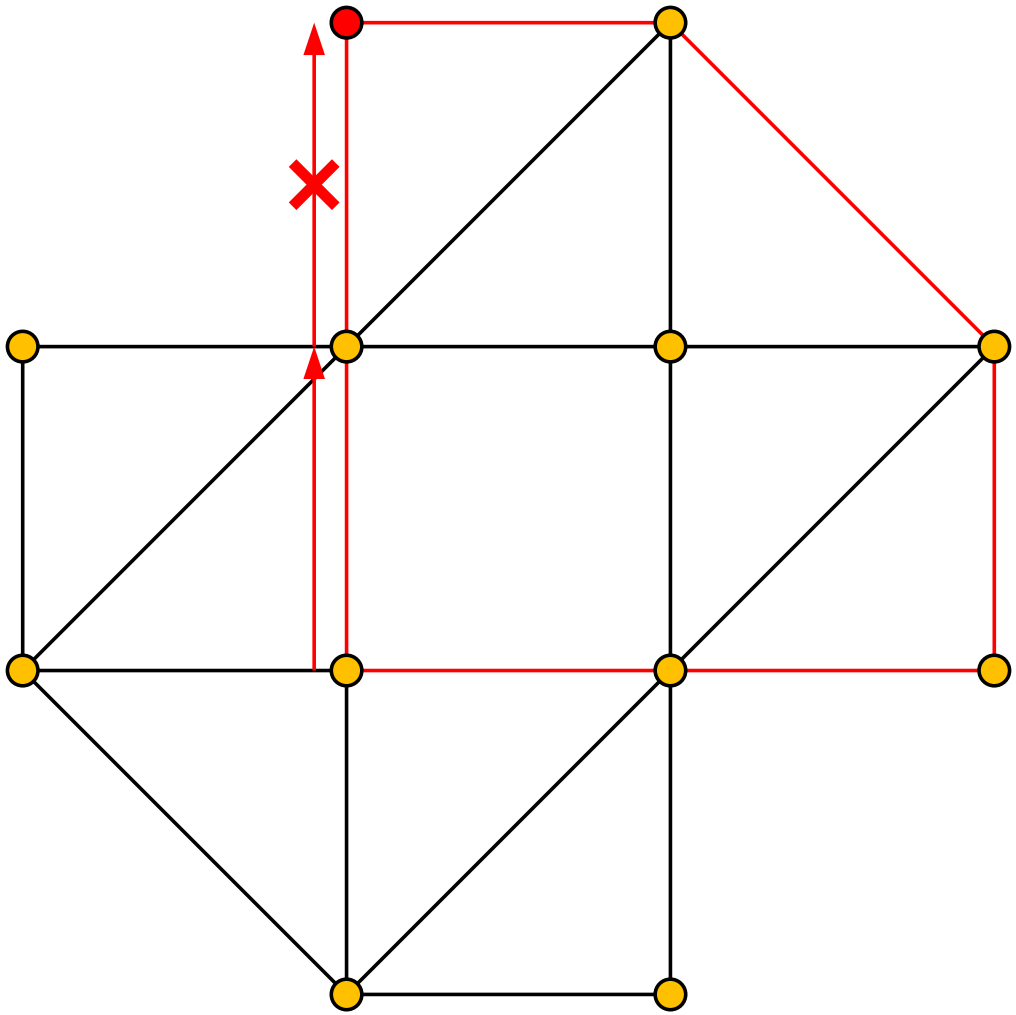

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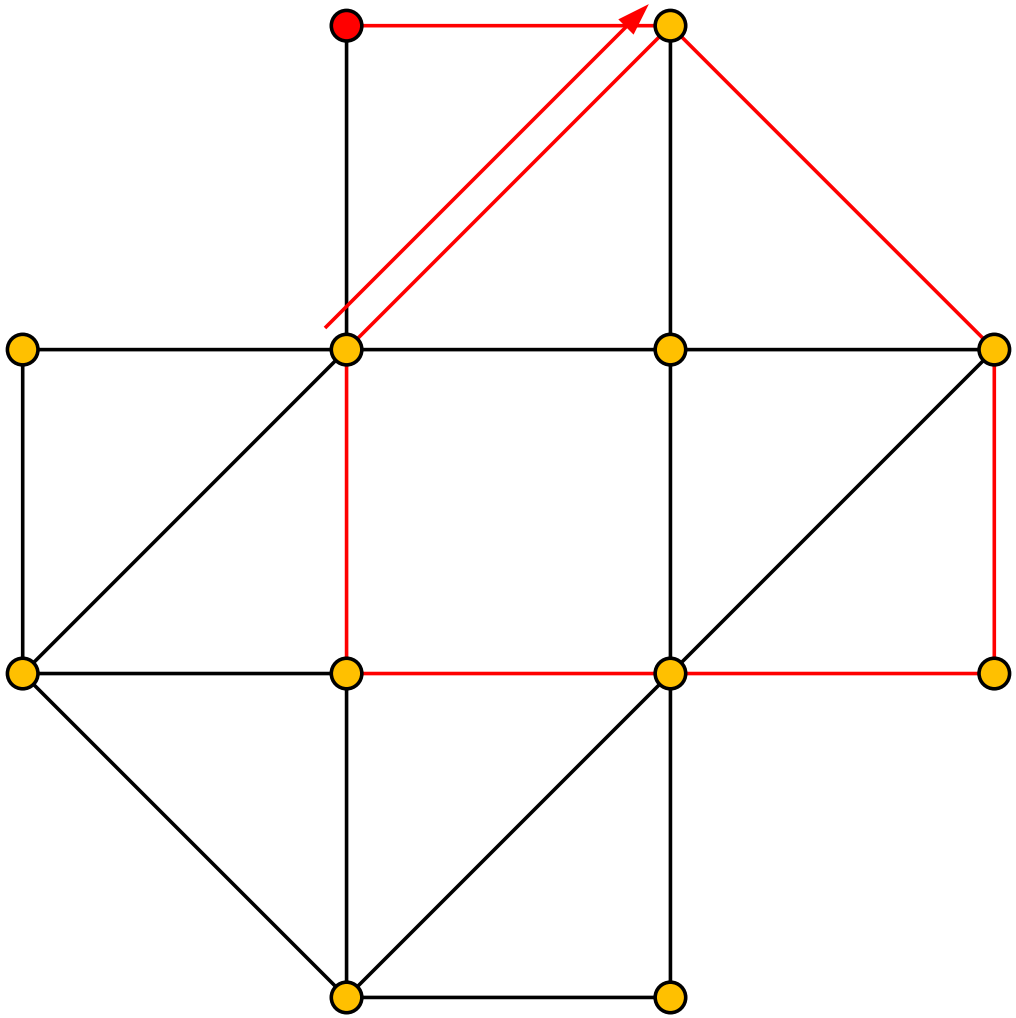

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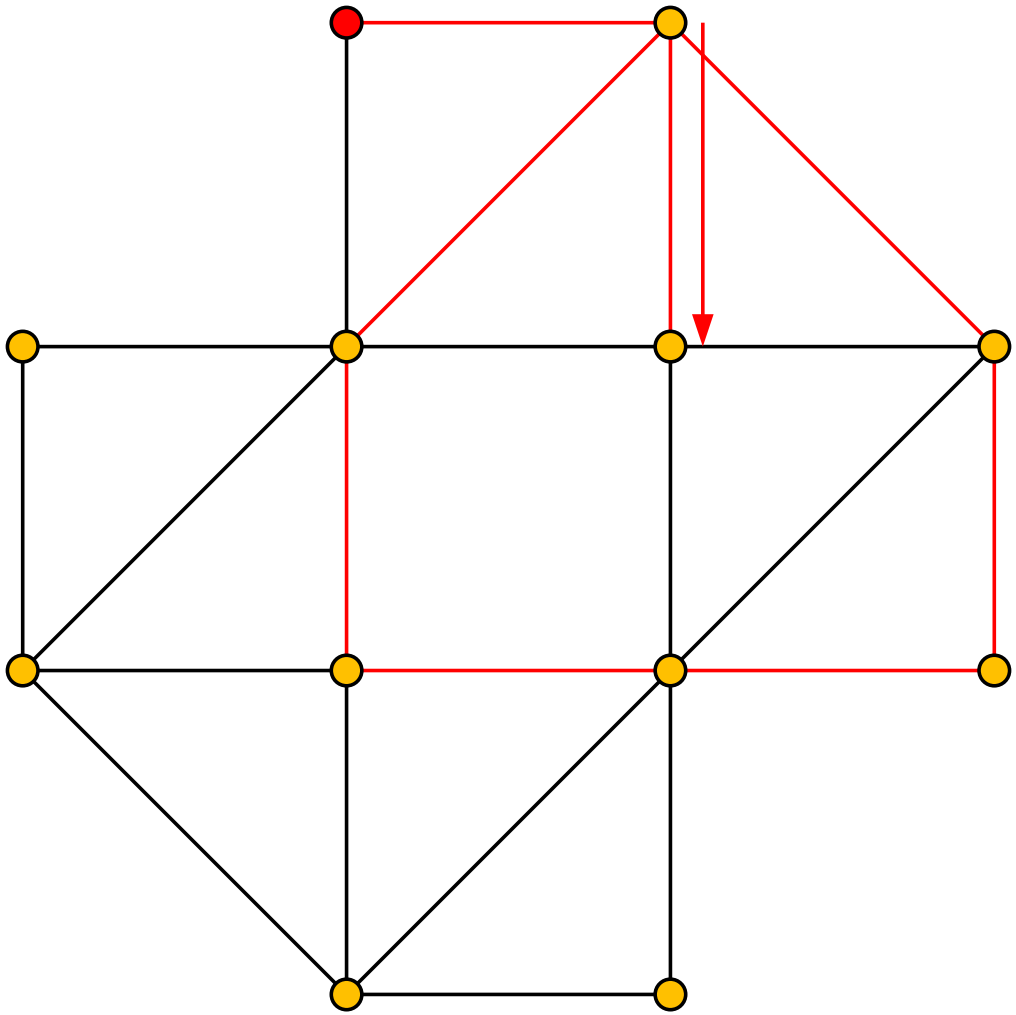

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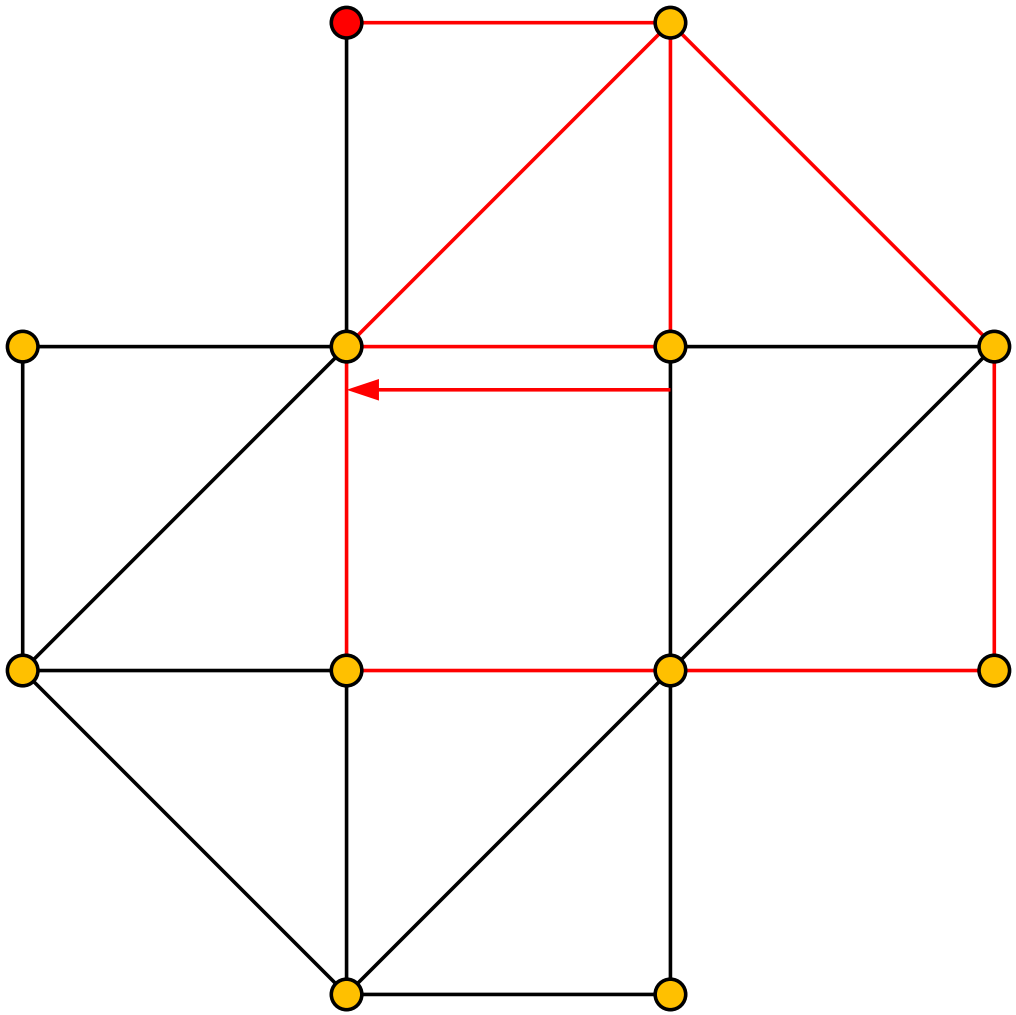

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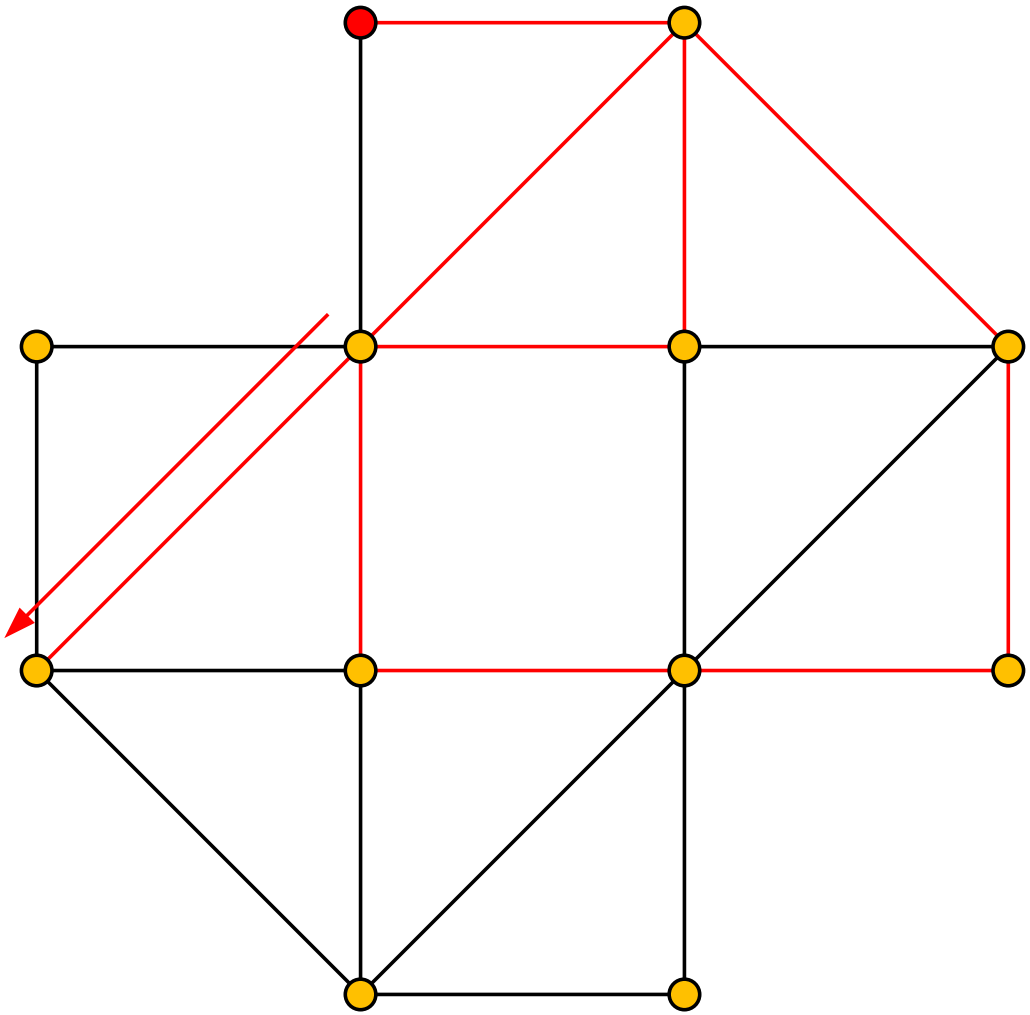

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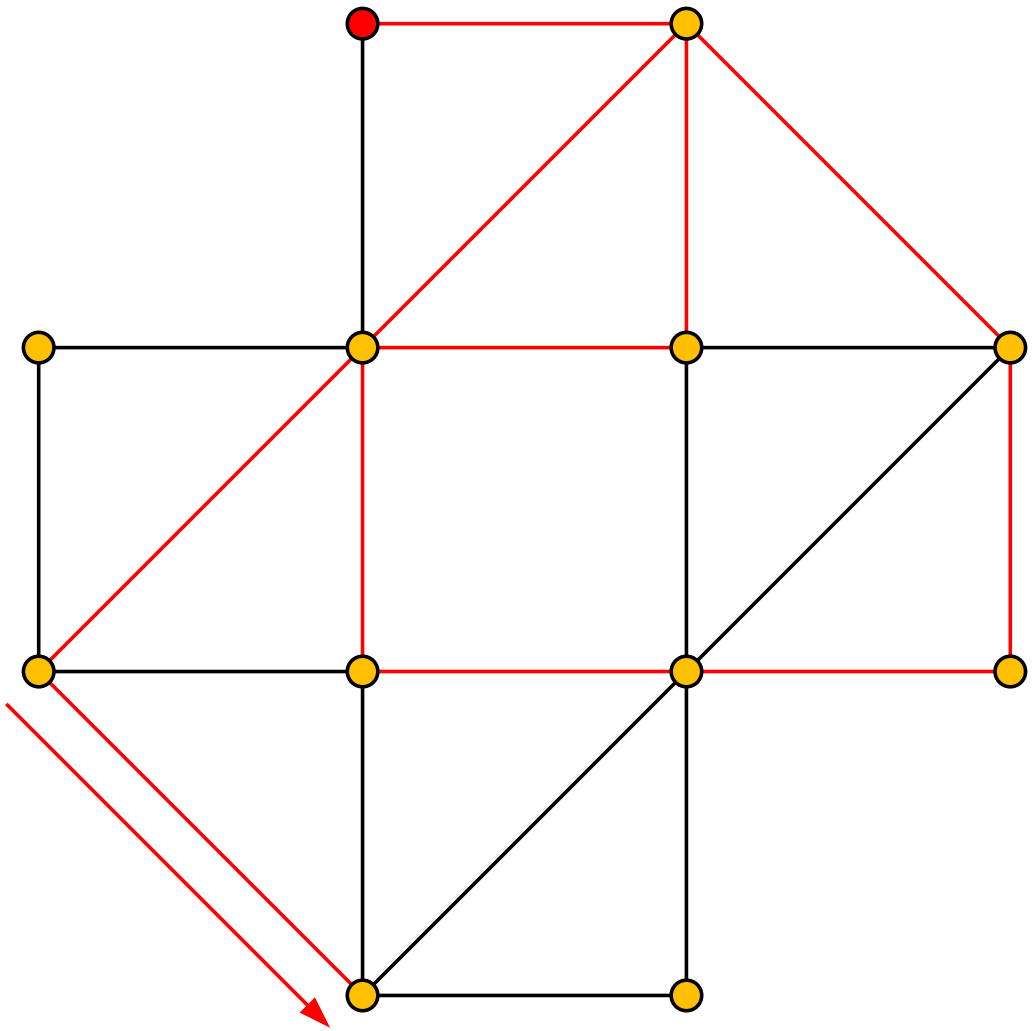

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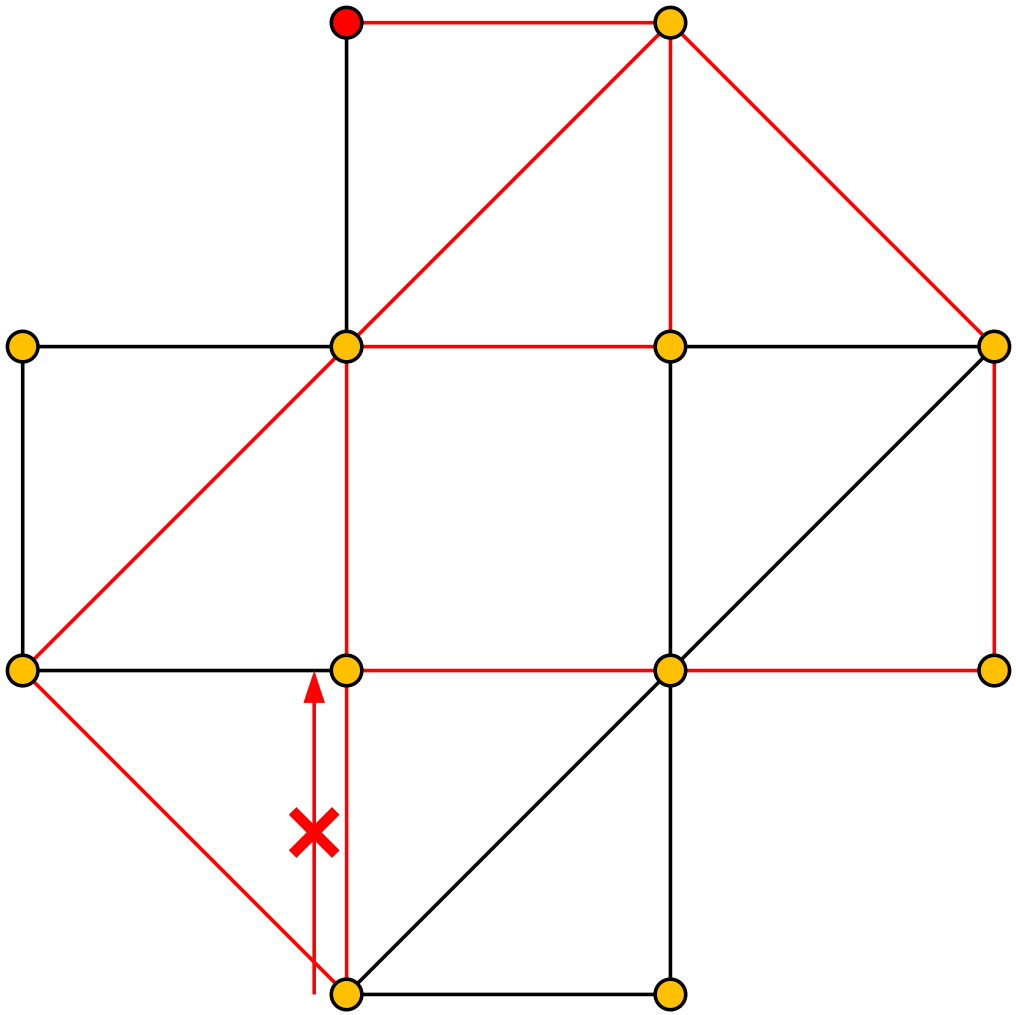

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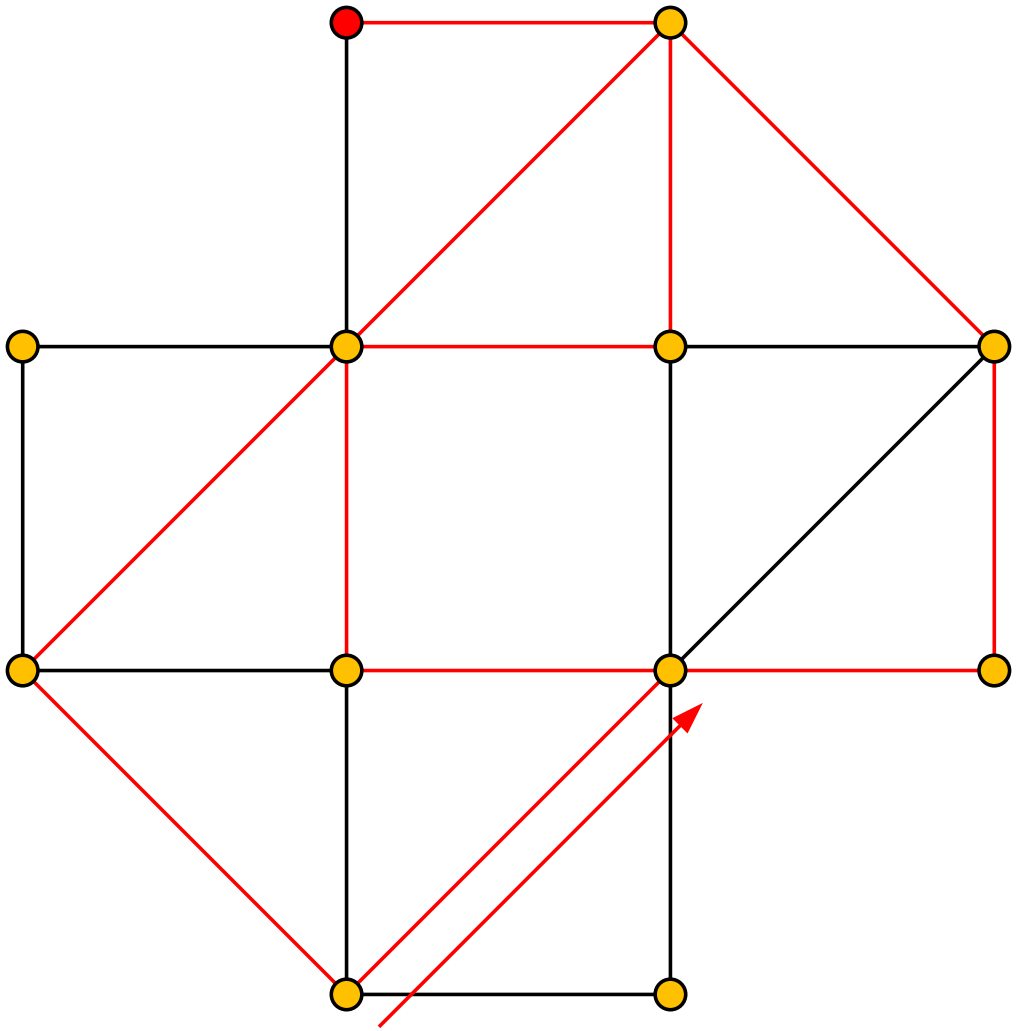

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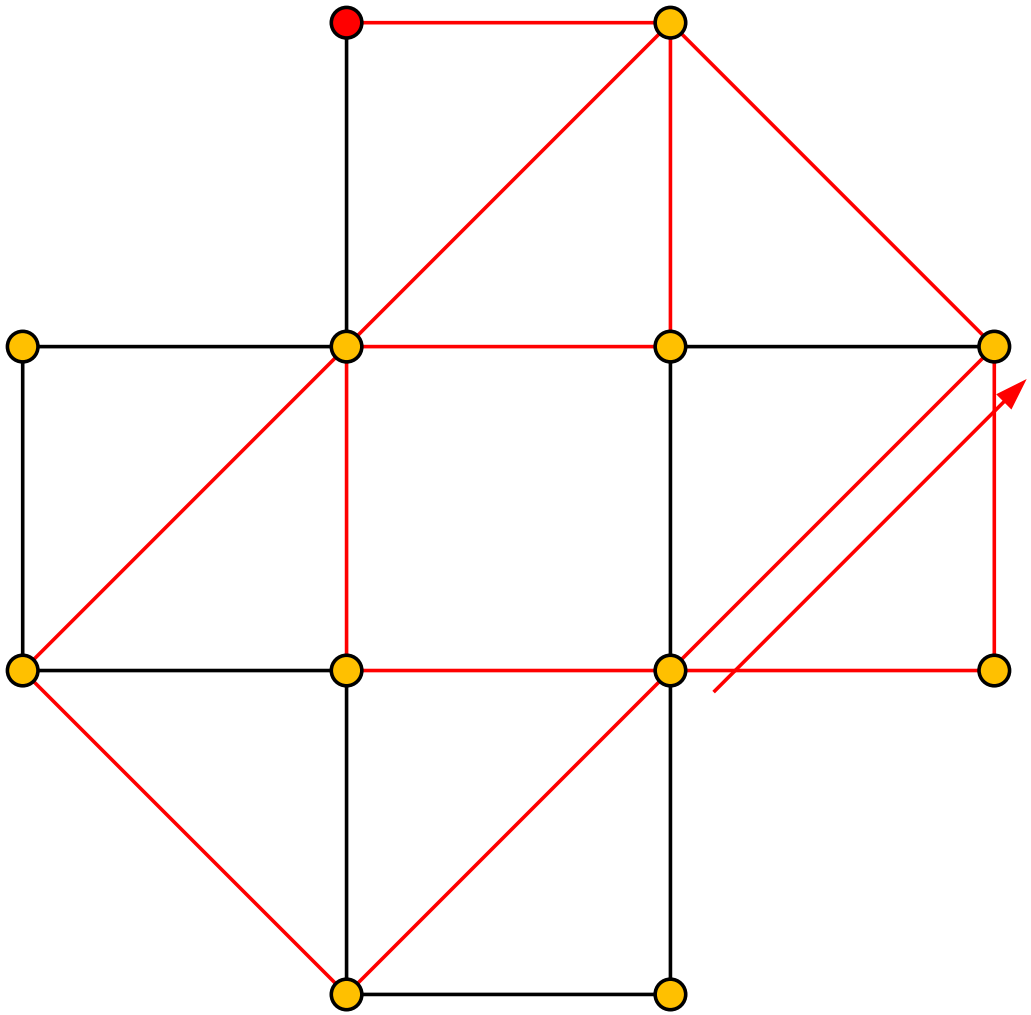

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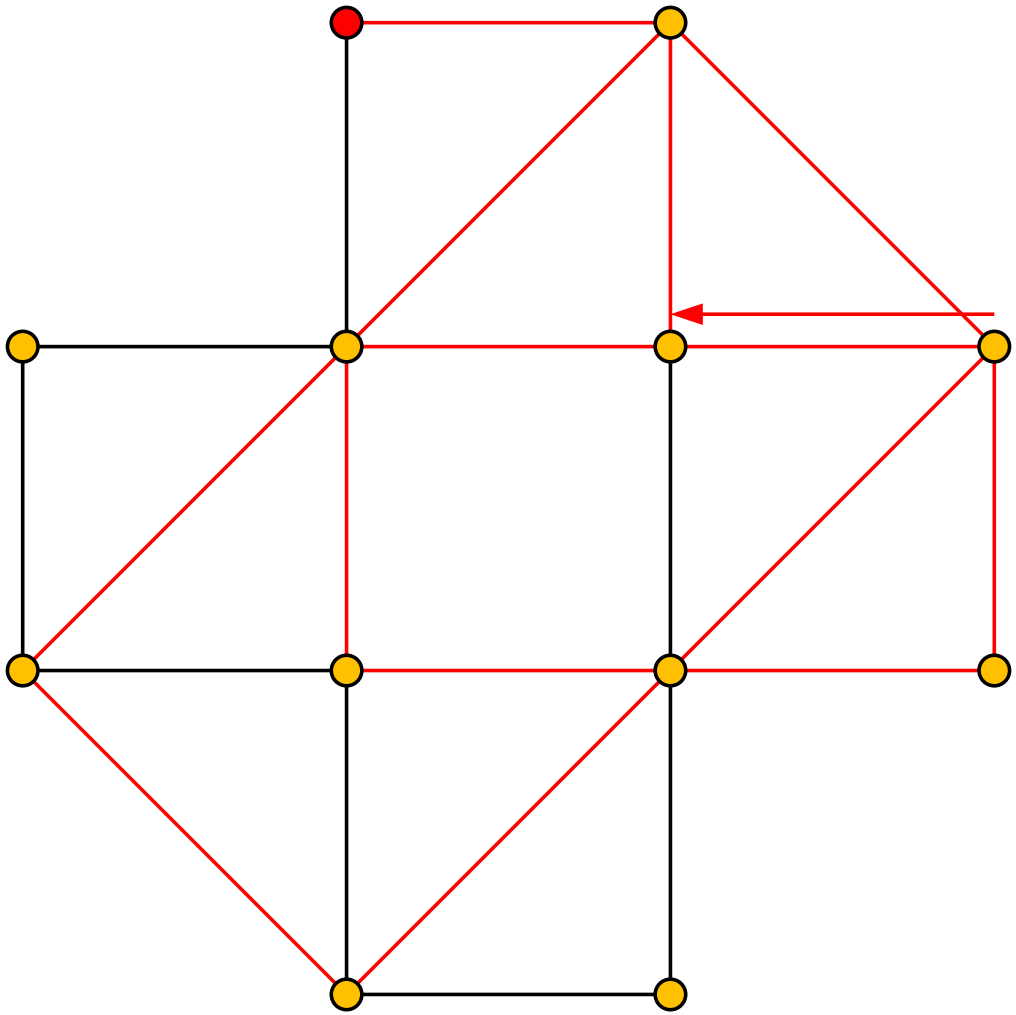

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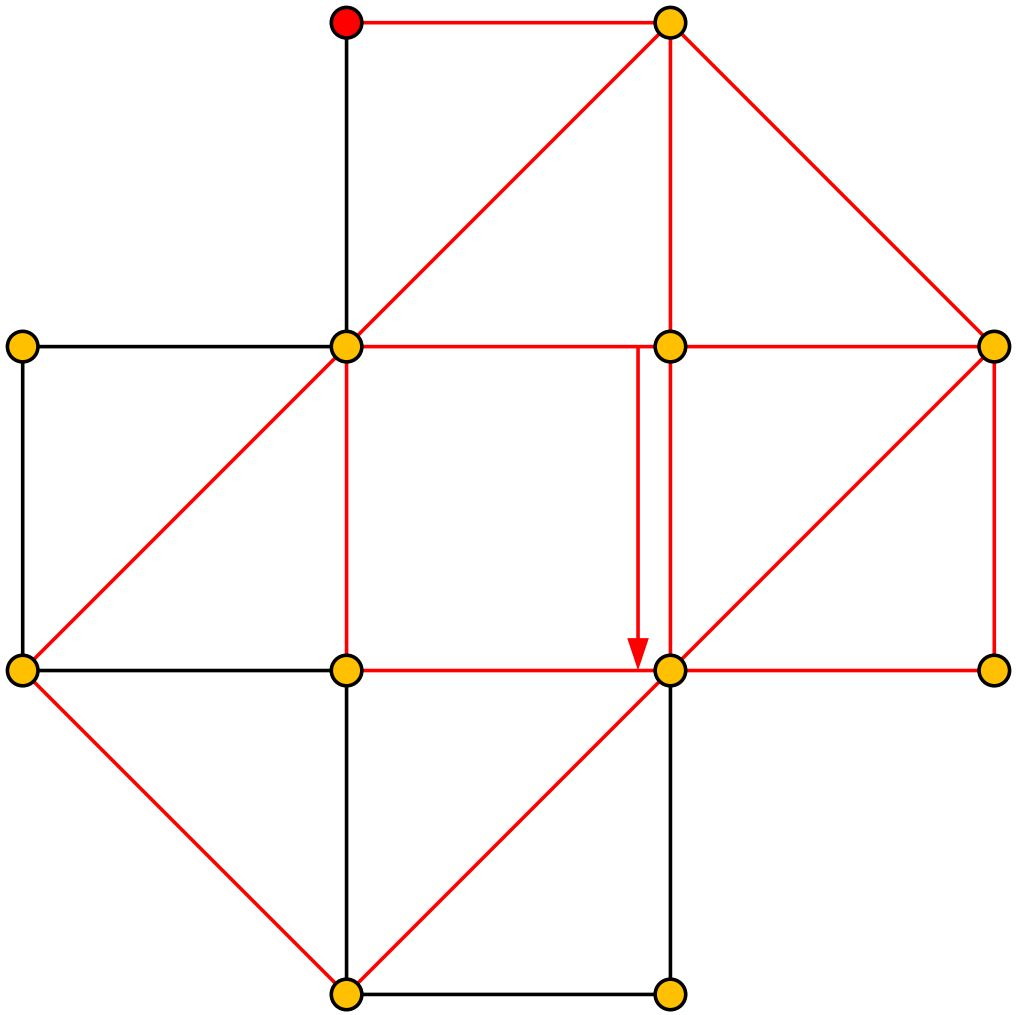

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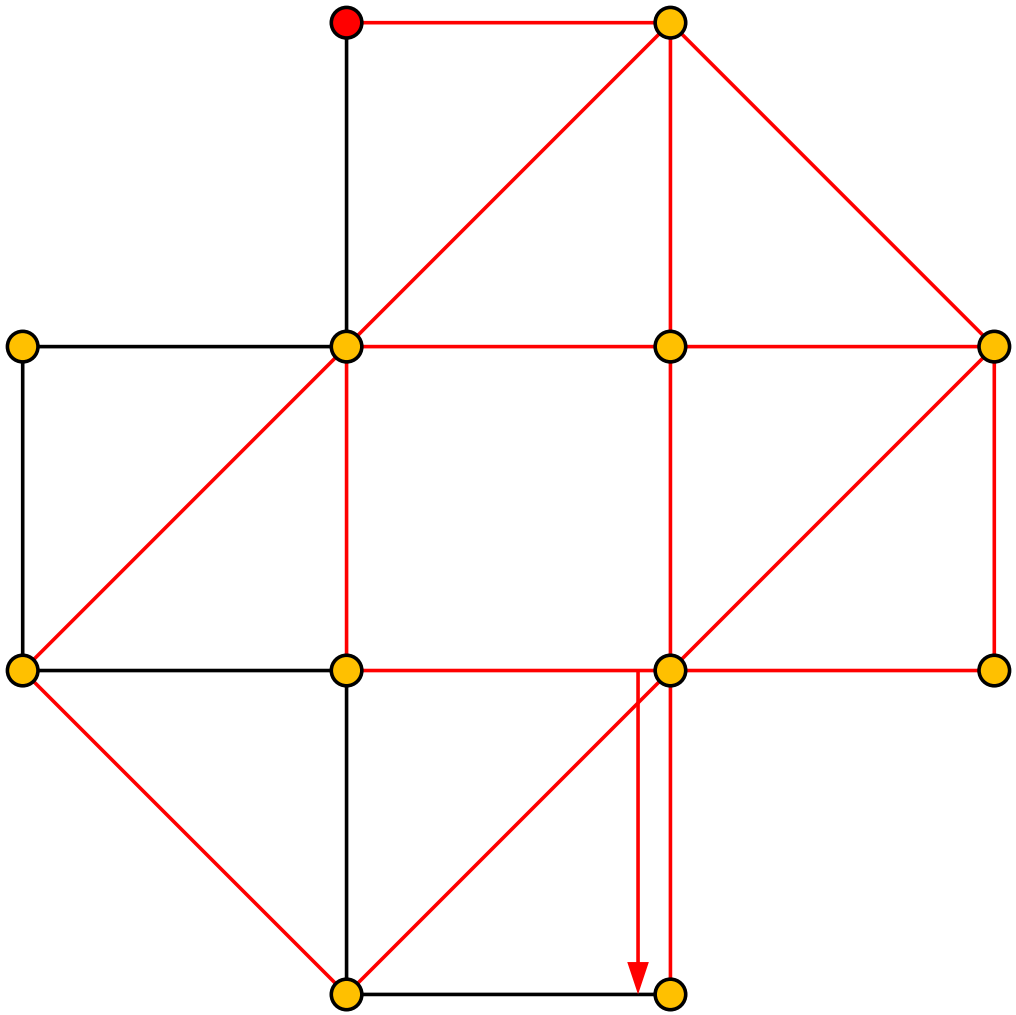

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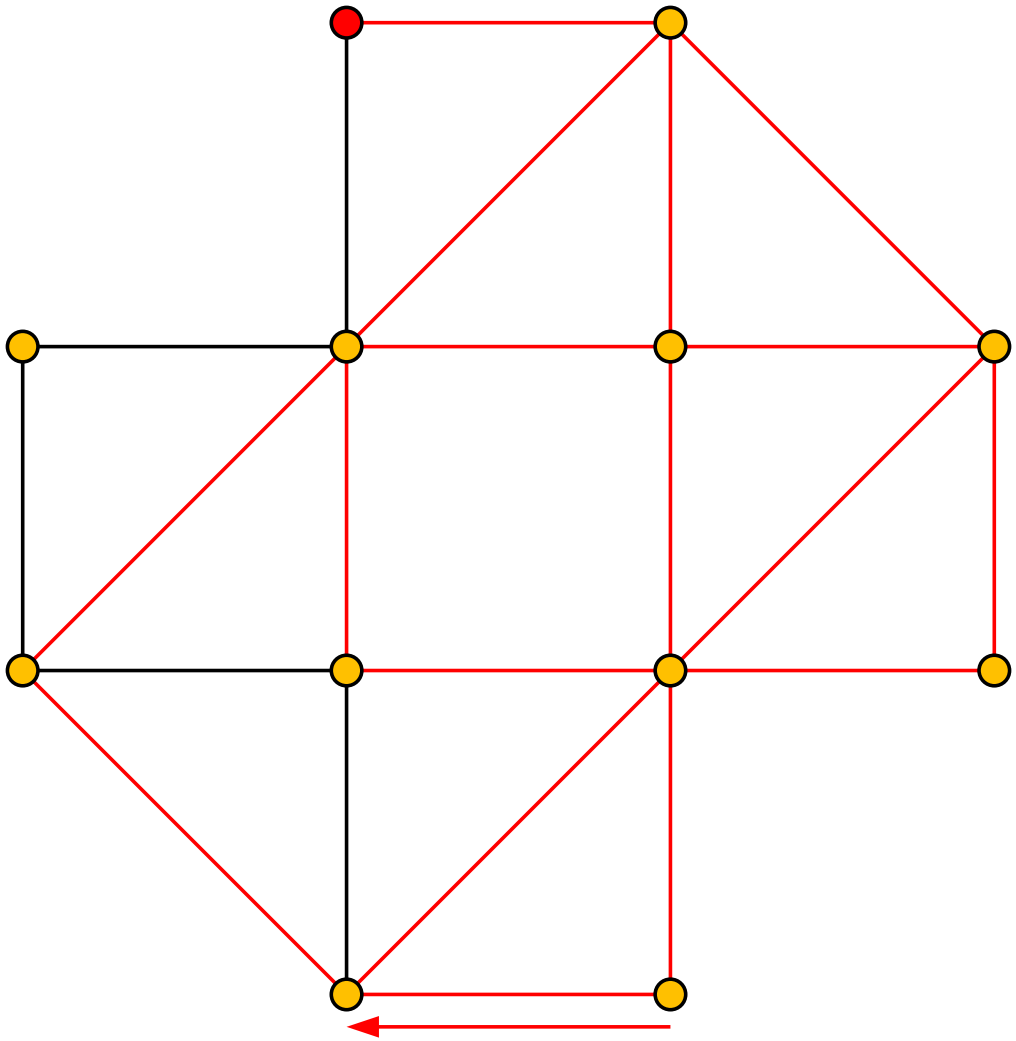

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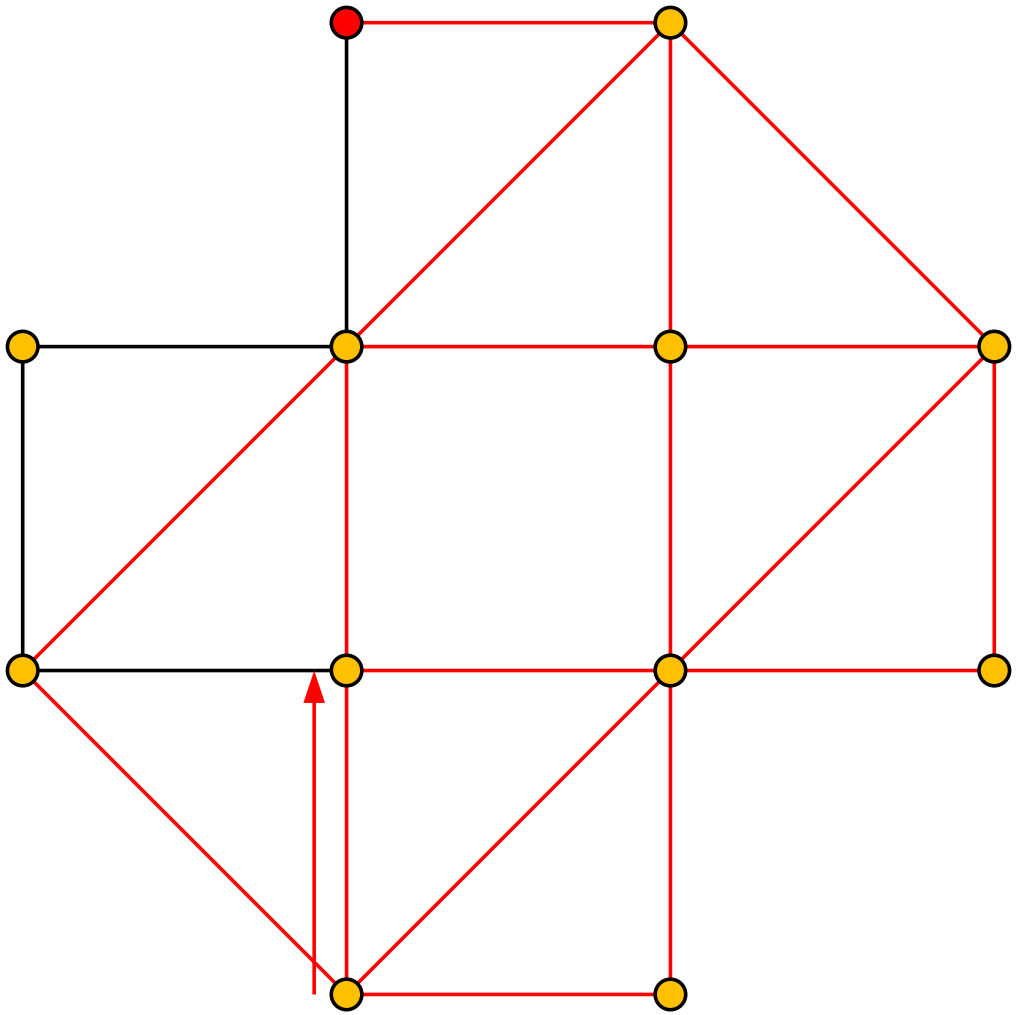

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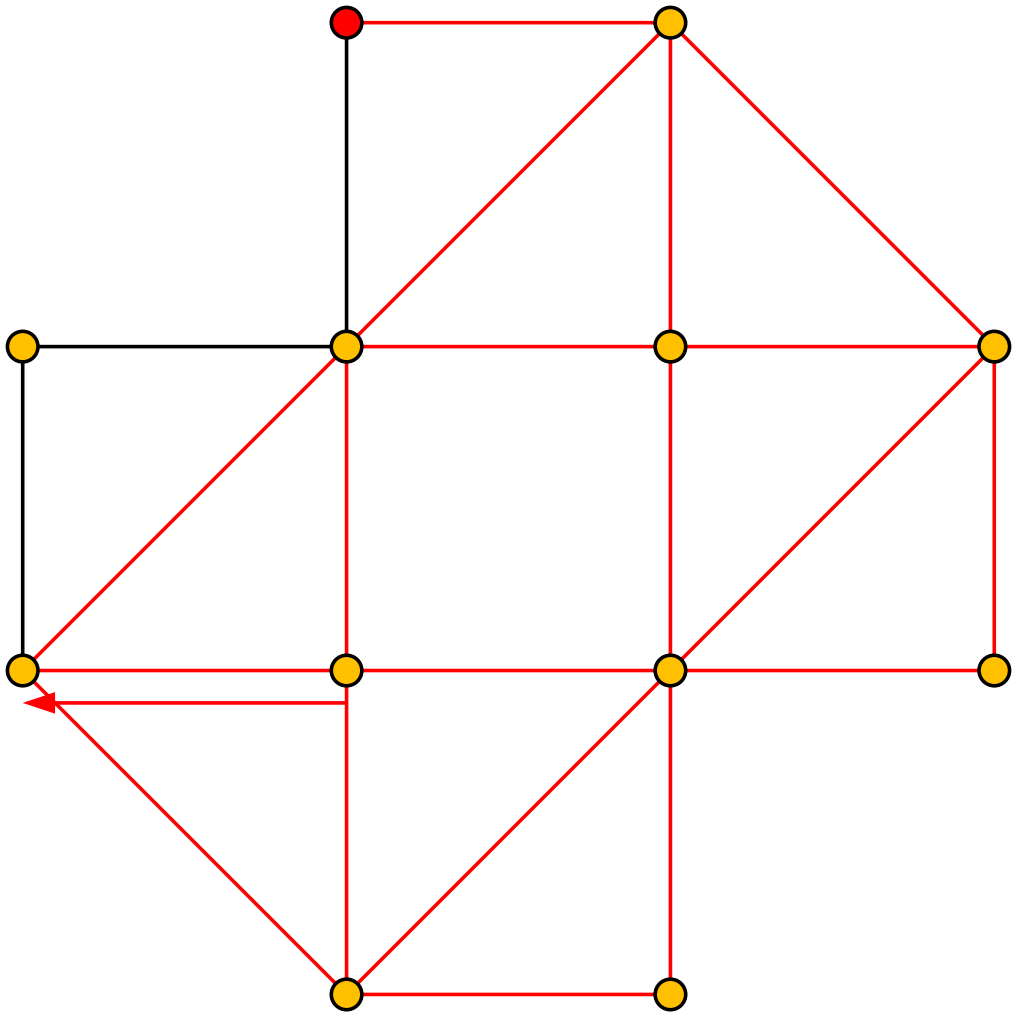

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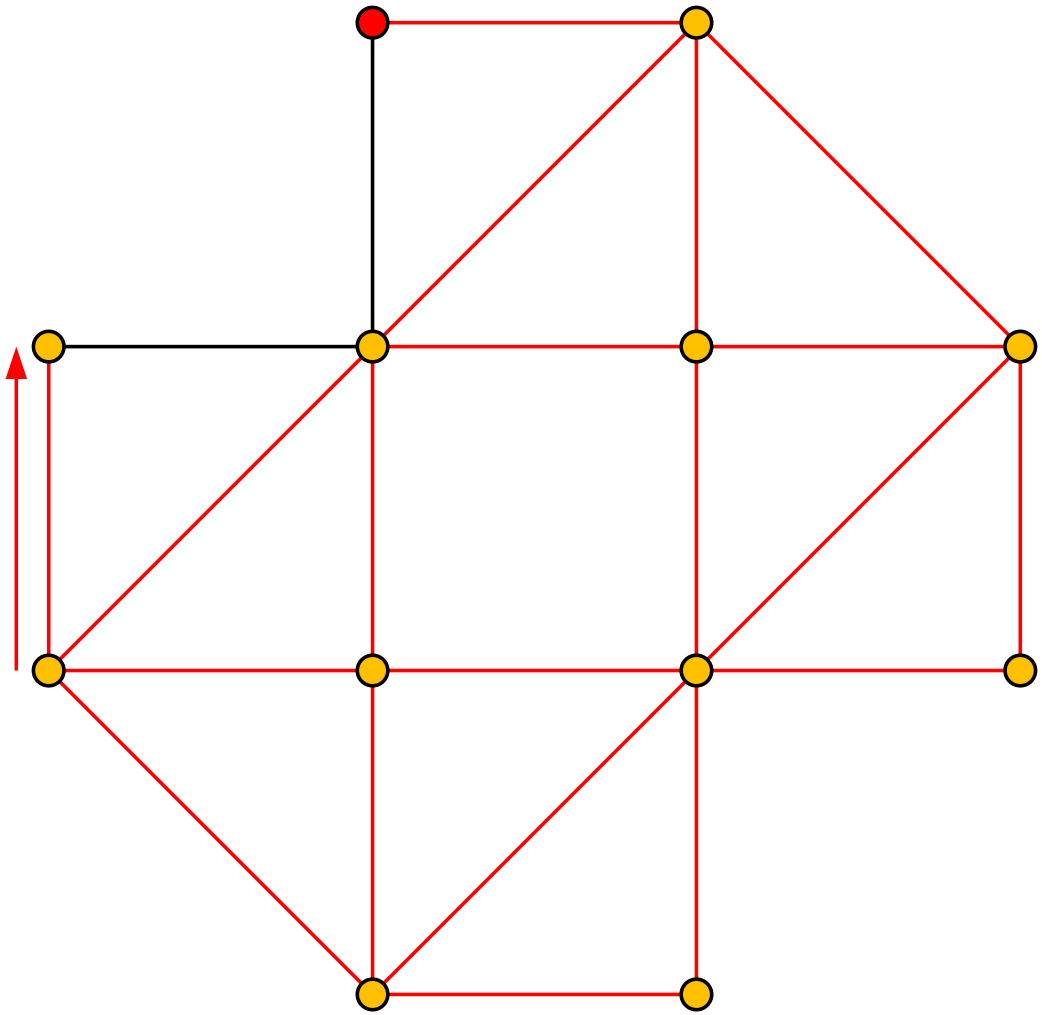

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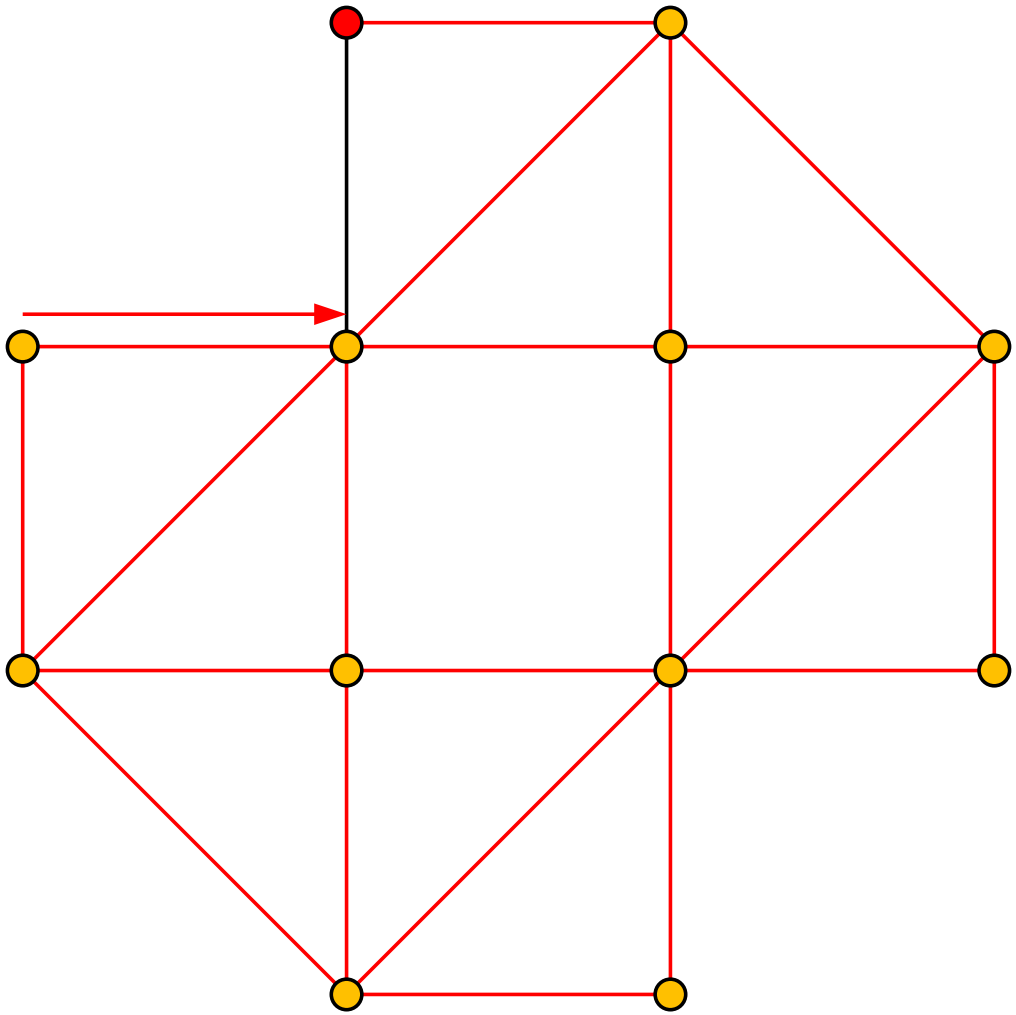

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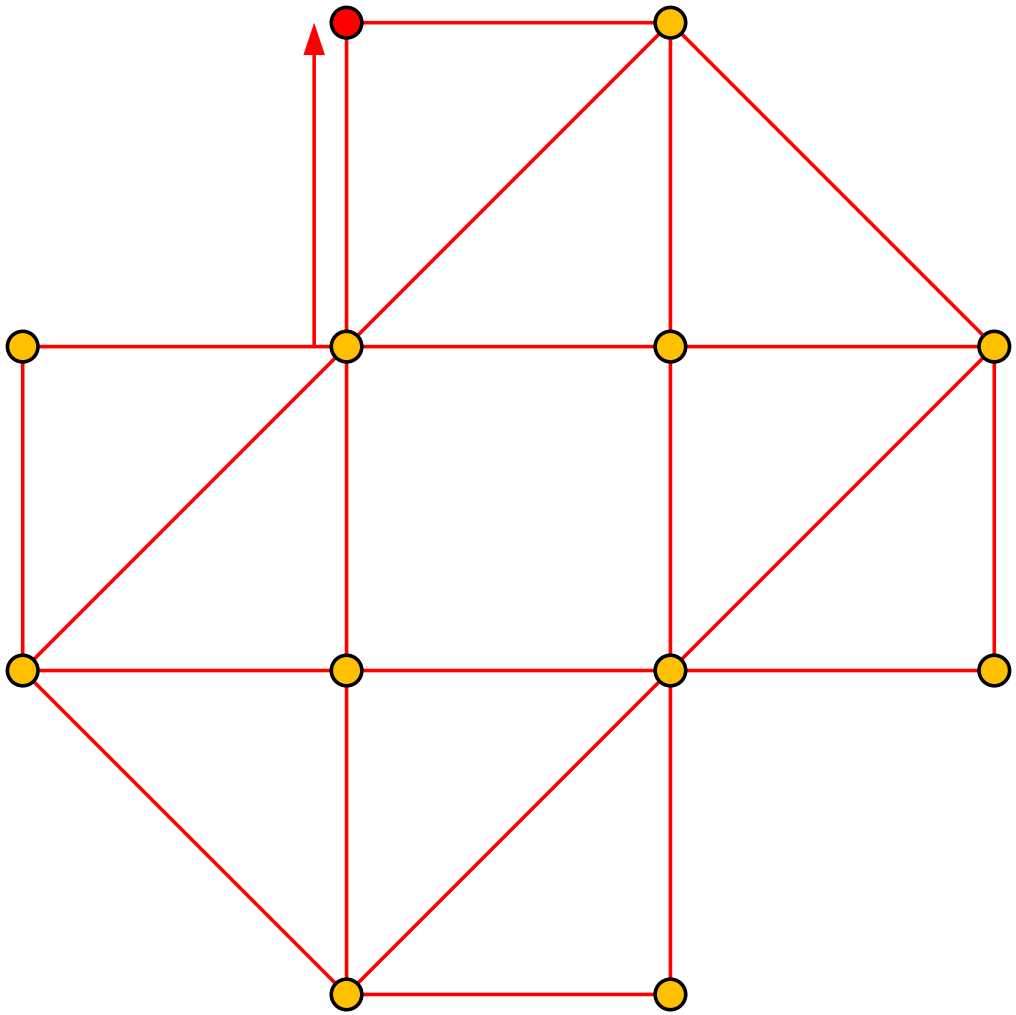

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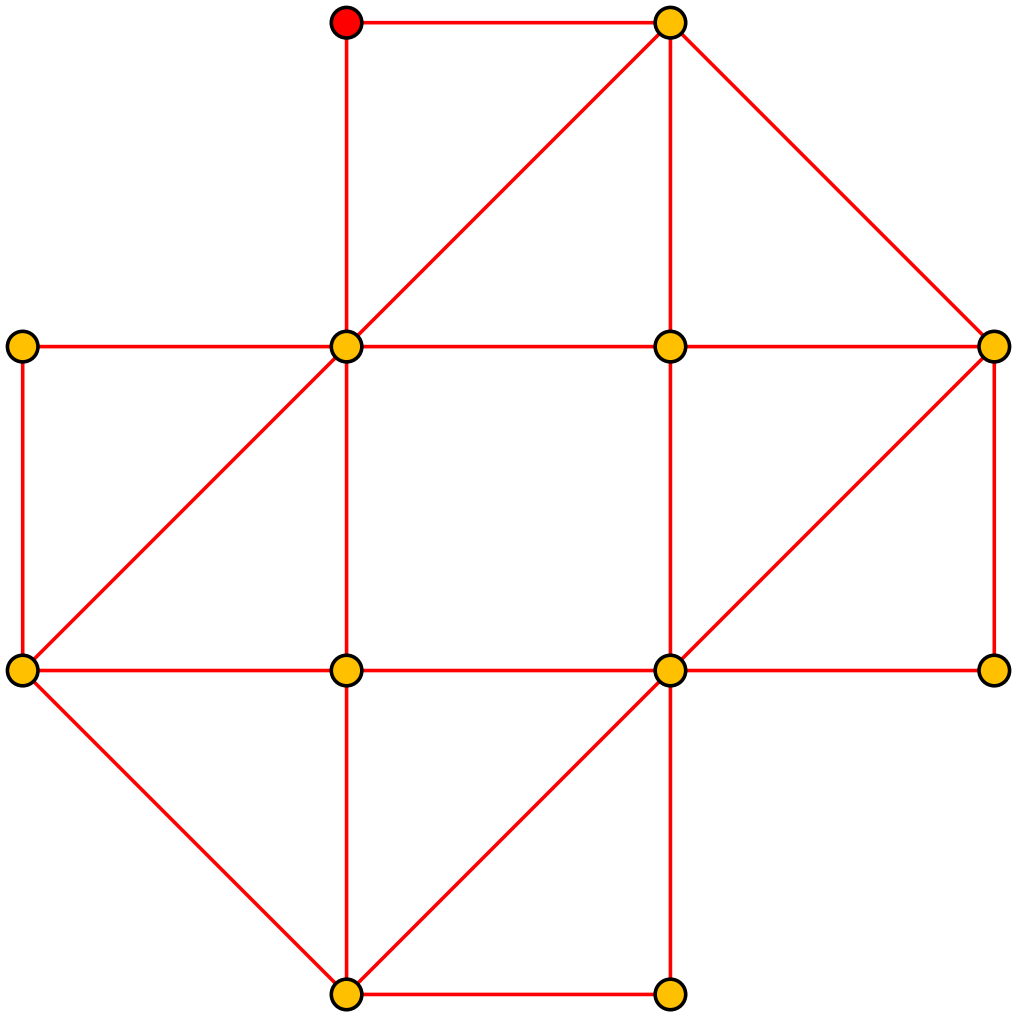

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- If there are untraveled edges, then retrace the circuit until you reach the first untraveled edge.

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- Follow untraveled edges, as before, until you return to that vertex.

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- “Splice” that circuit into the original circuit.
- Repeat the last three steps until there are no more untraveled edges. You now have an Euler circuit.

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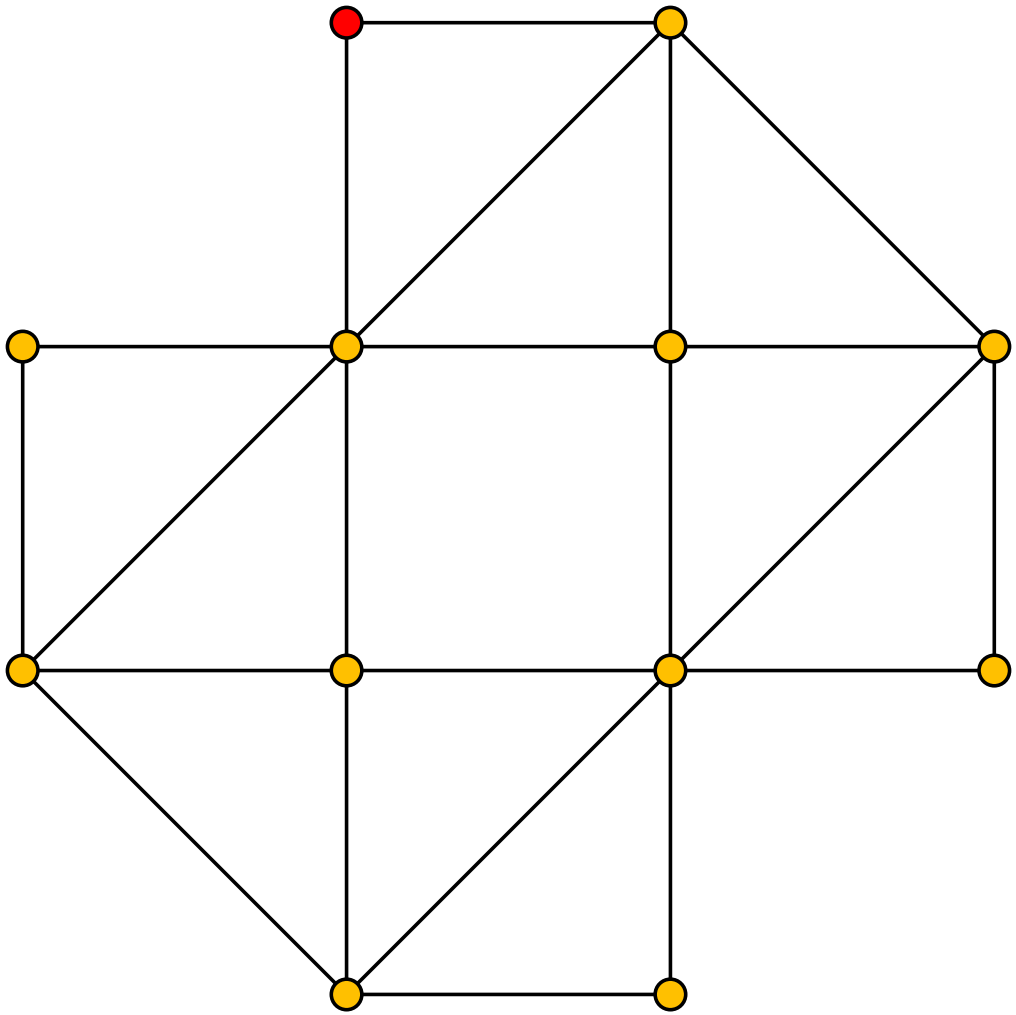

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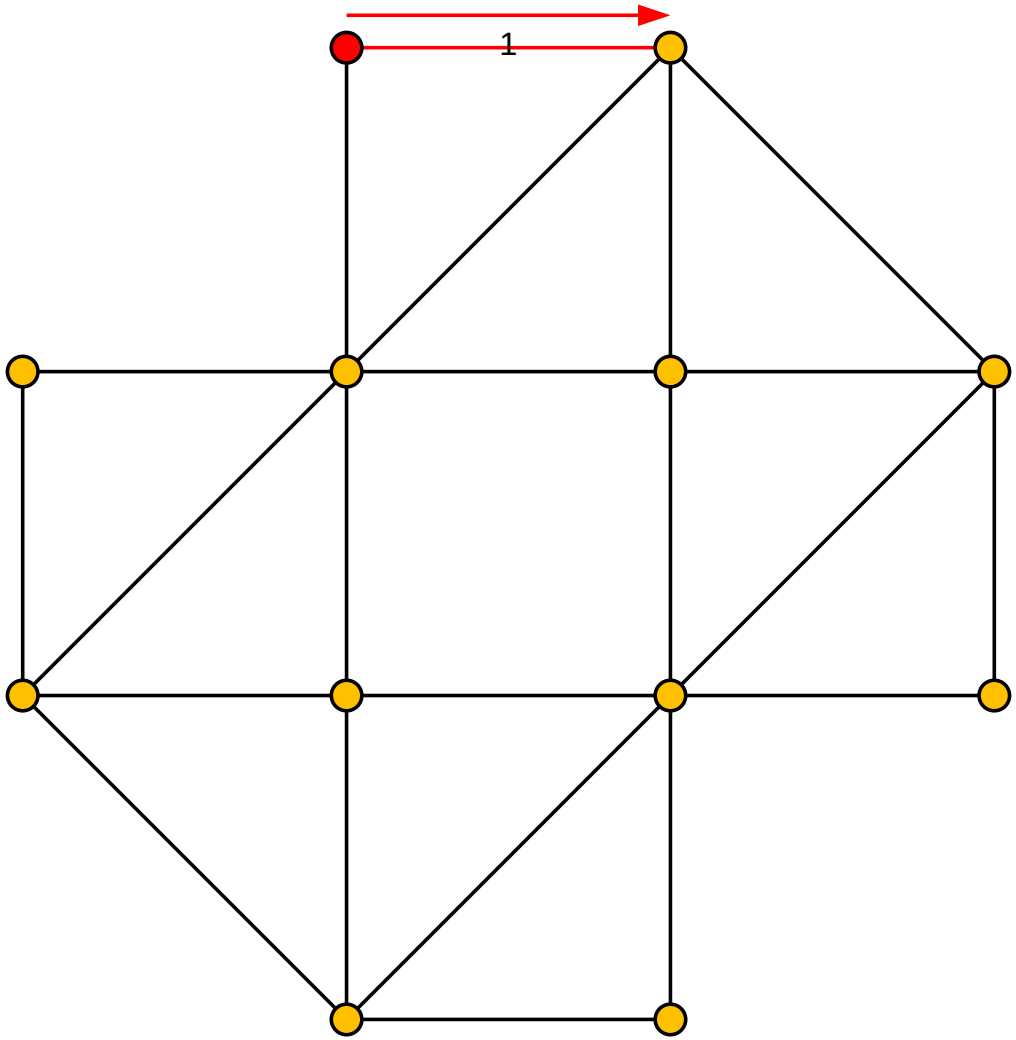

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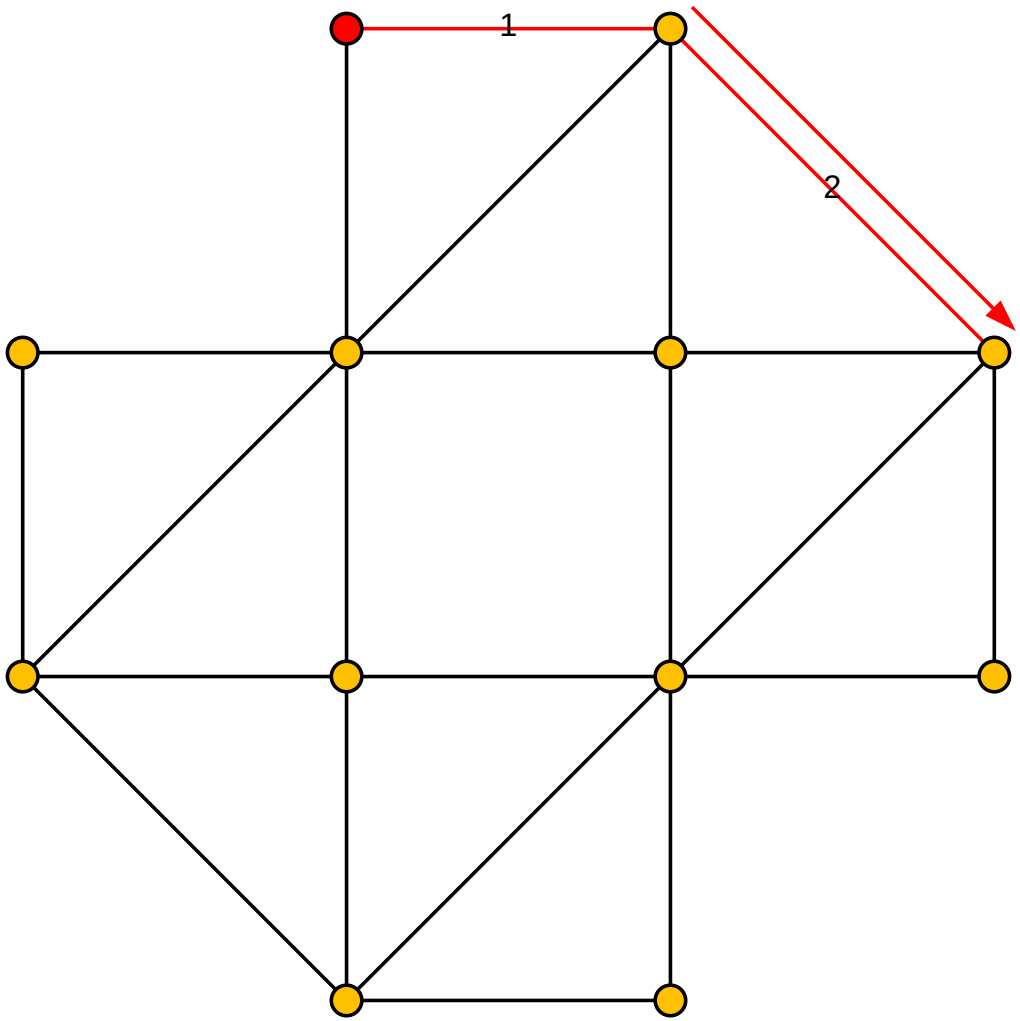

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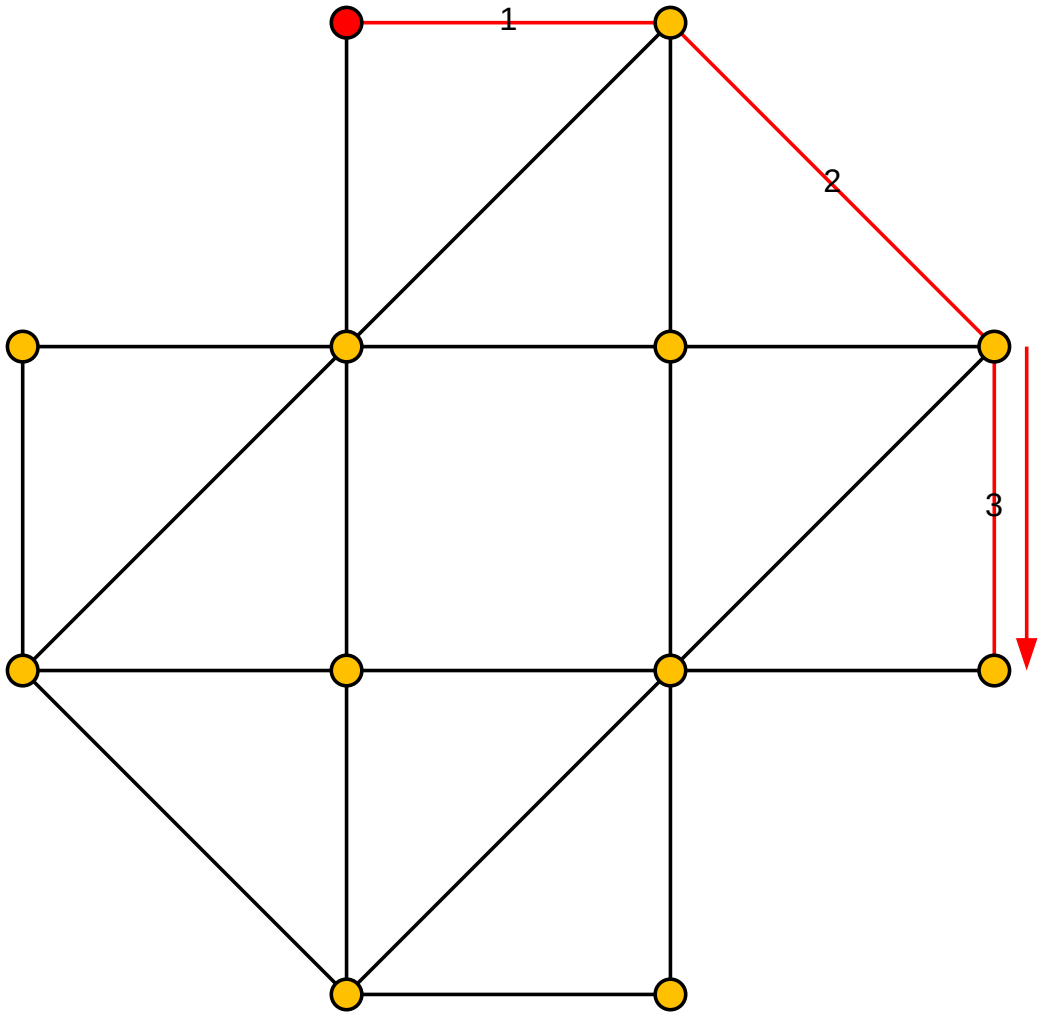

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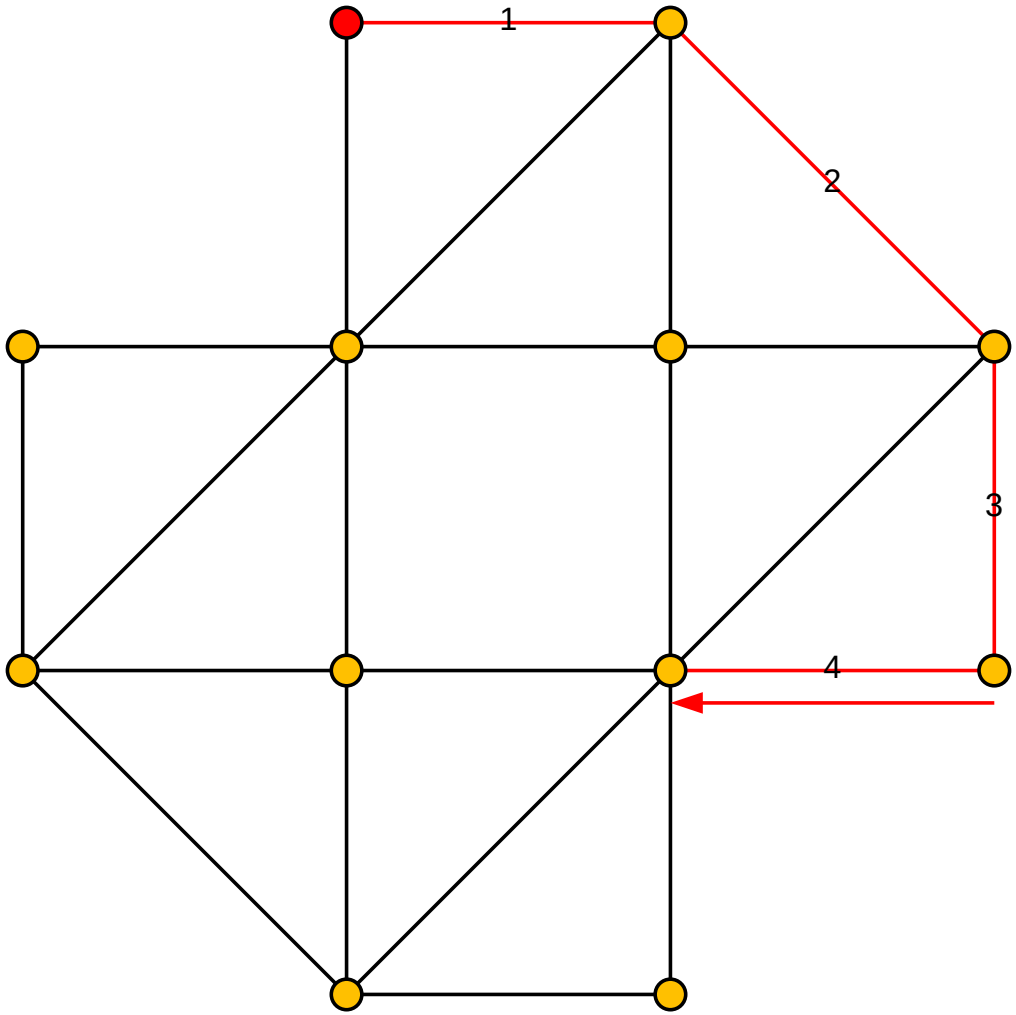

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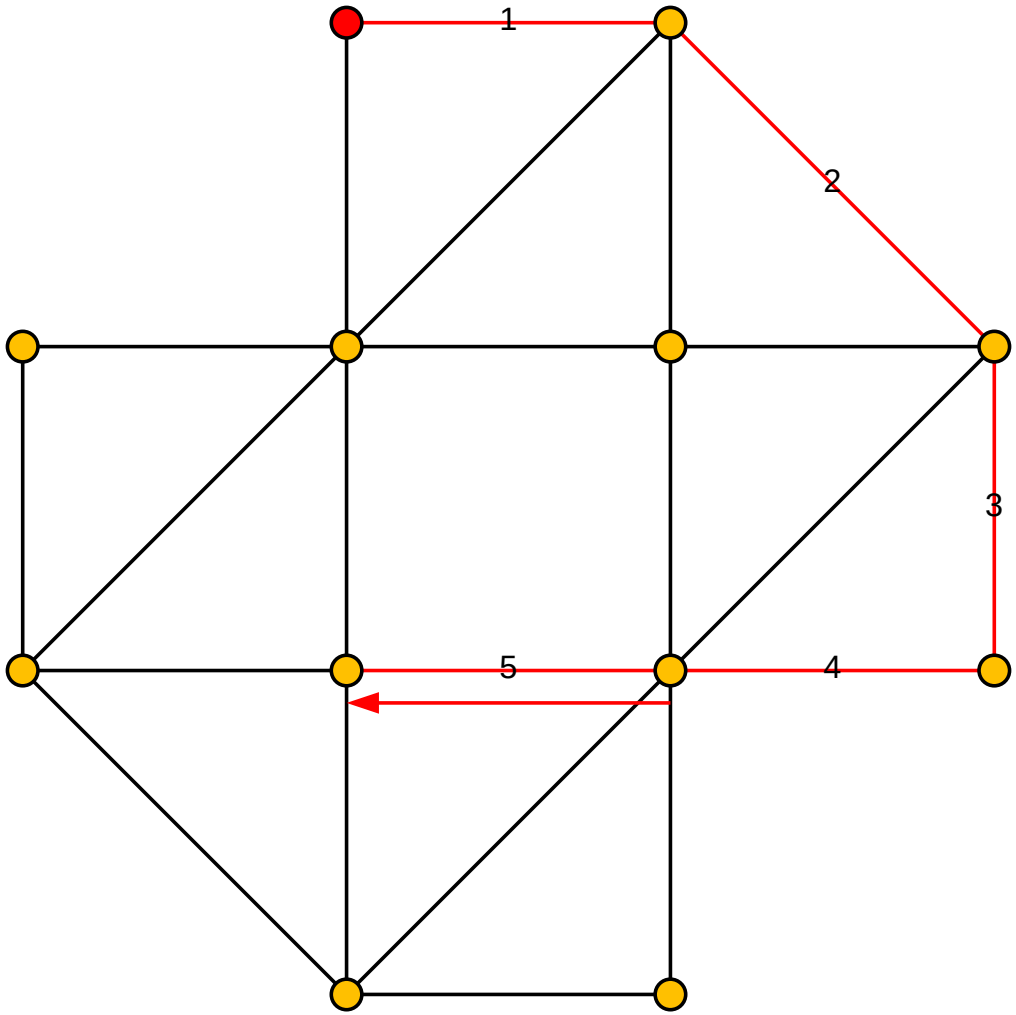

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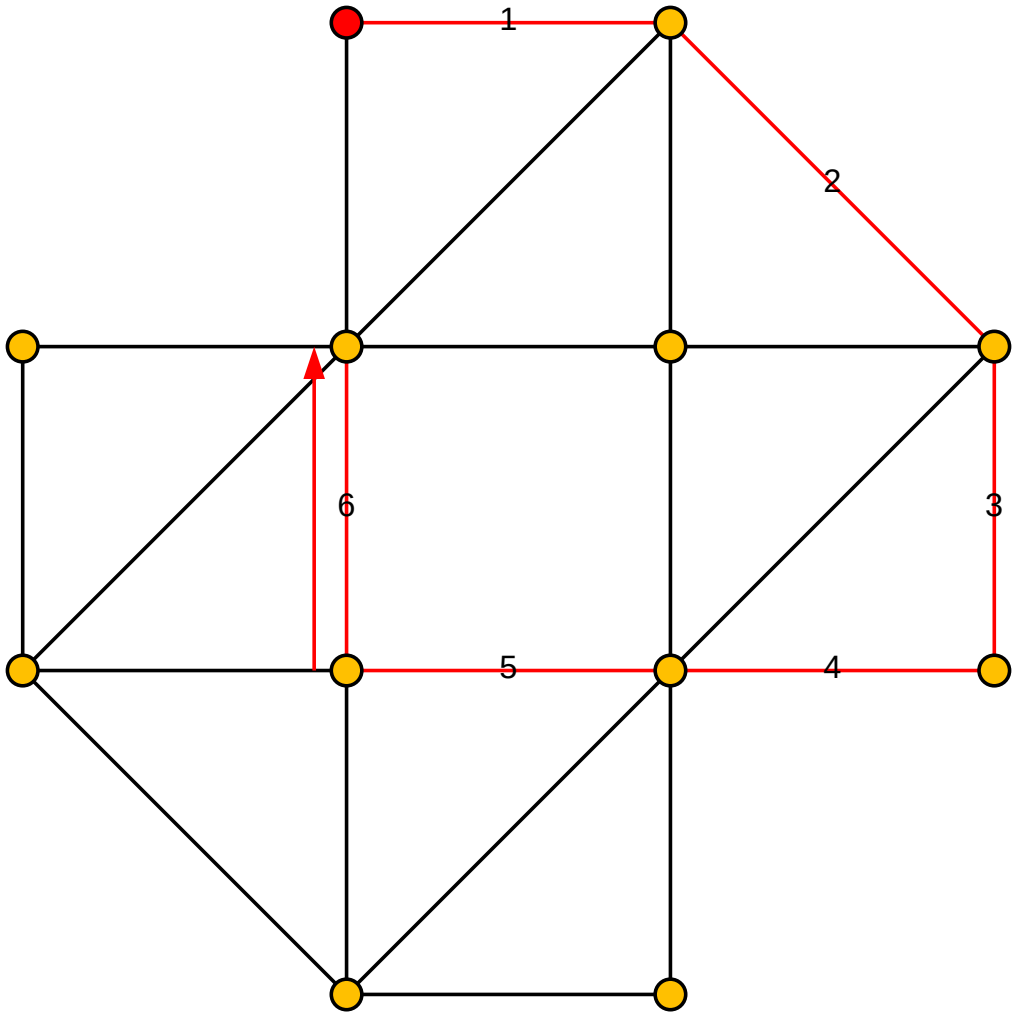

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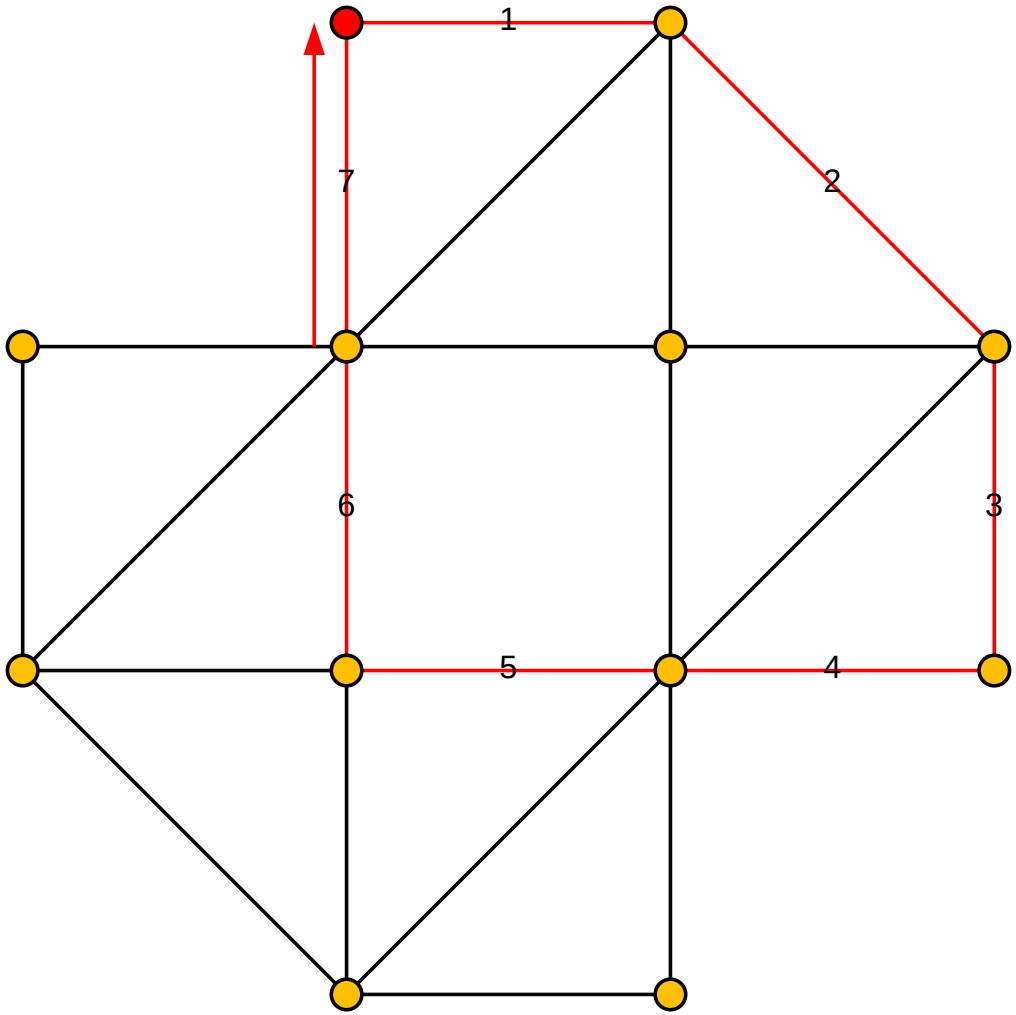

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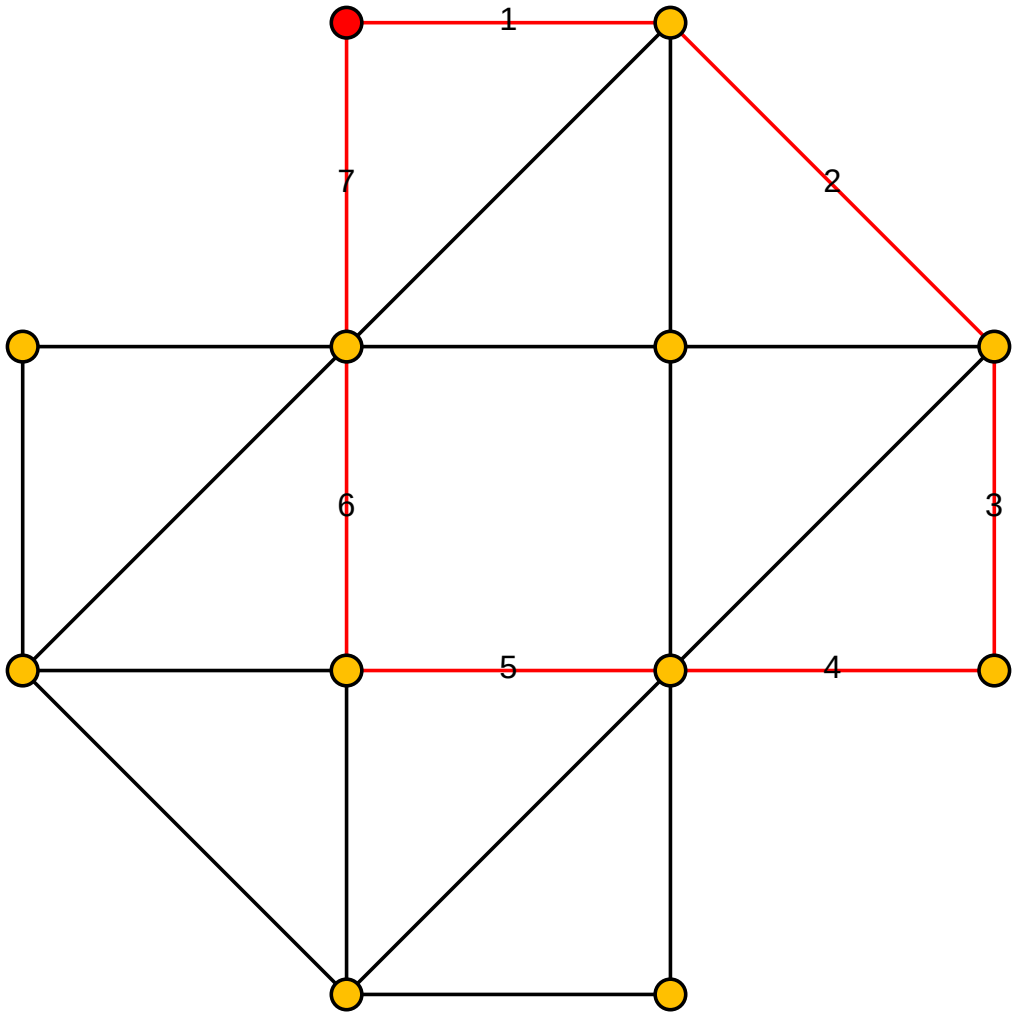

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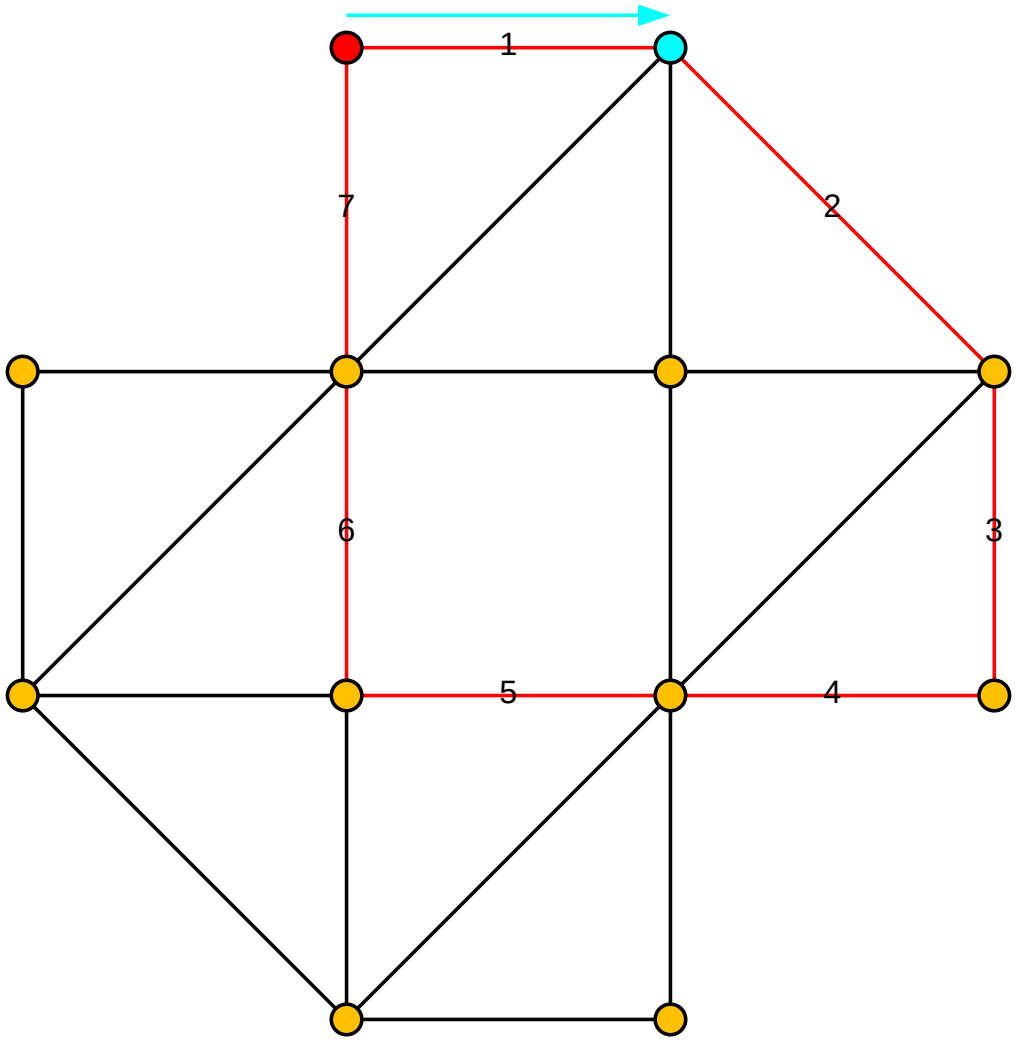

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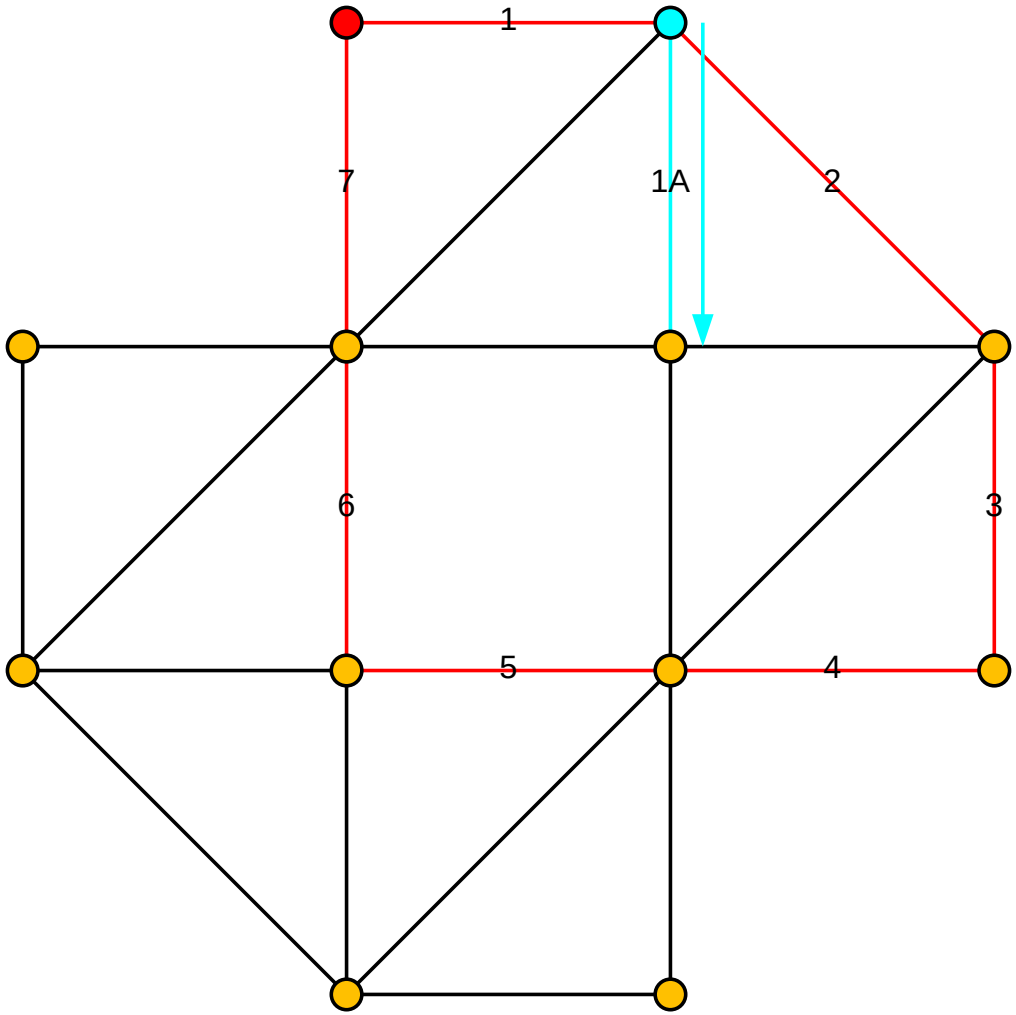

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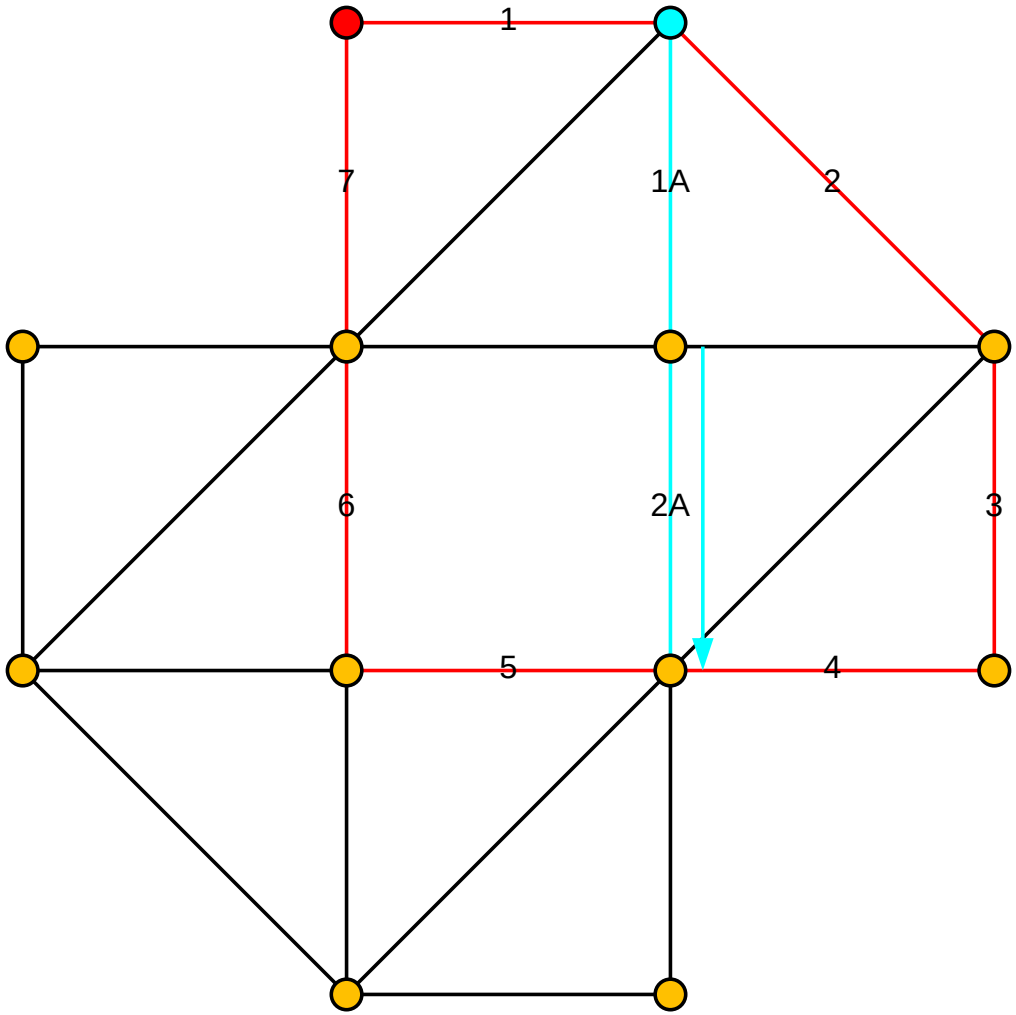

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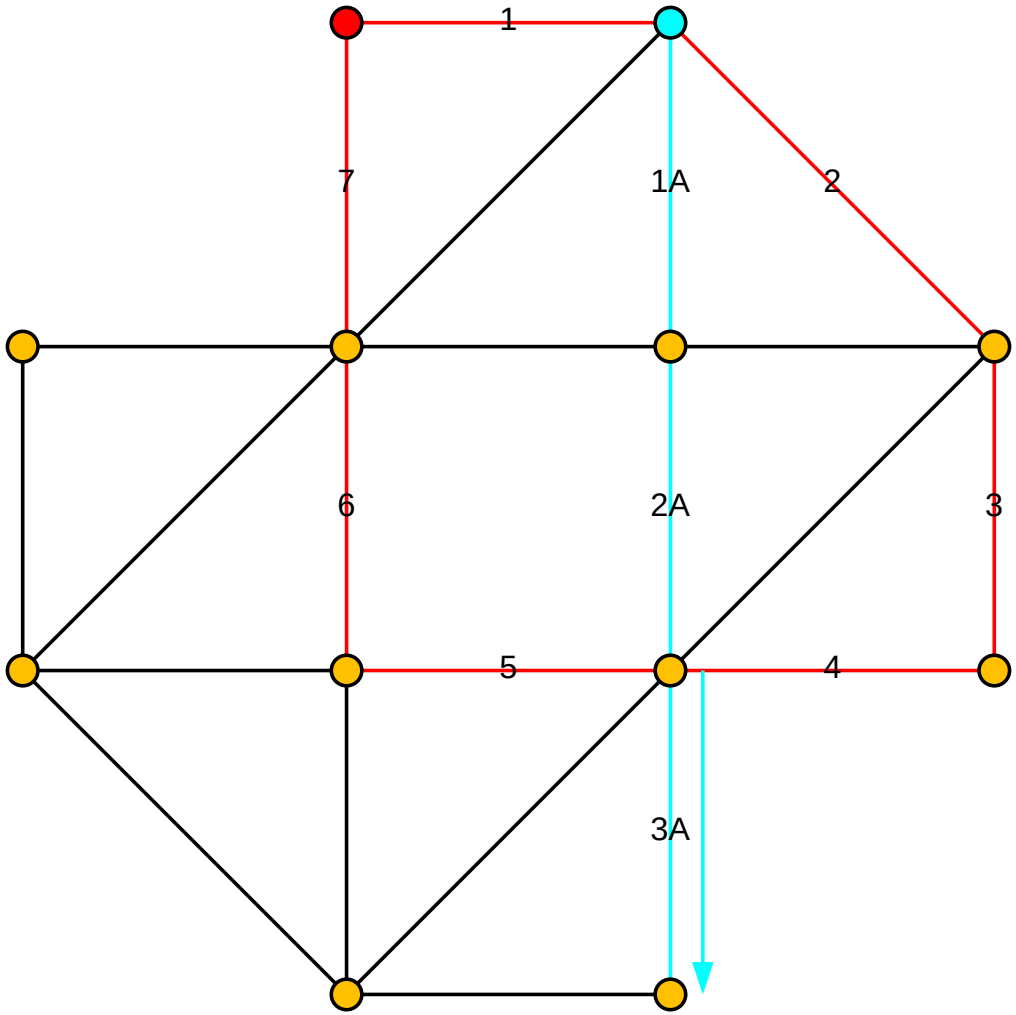

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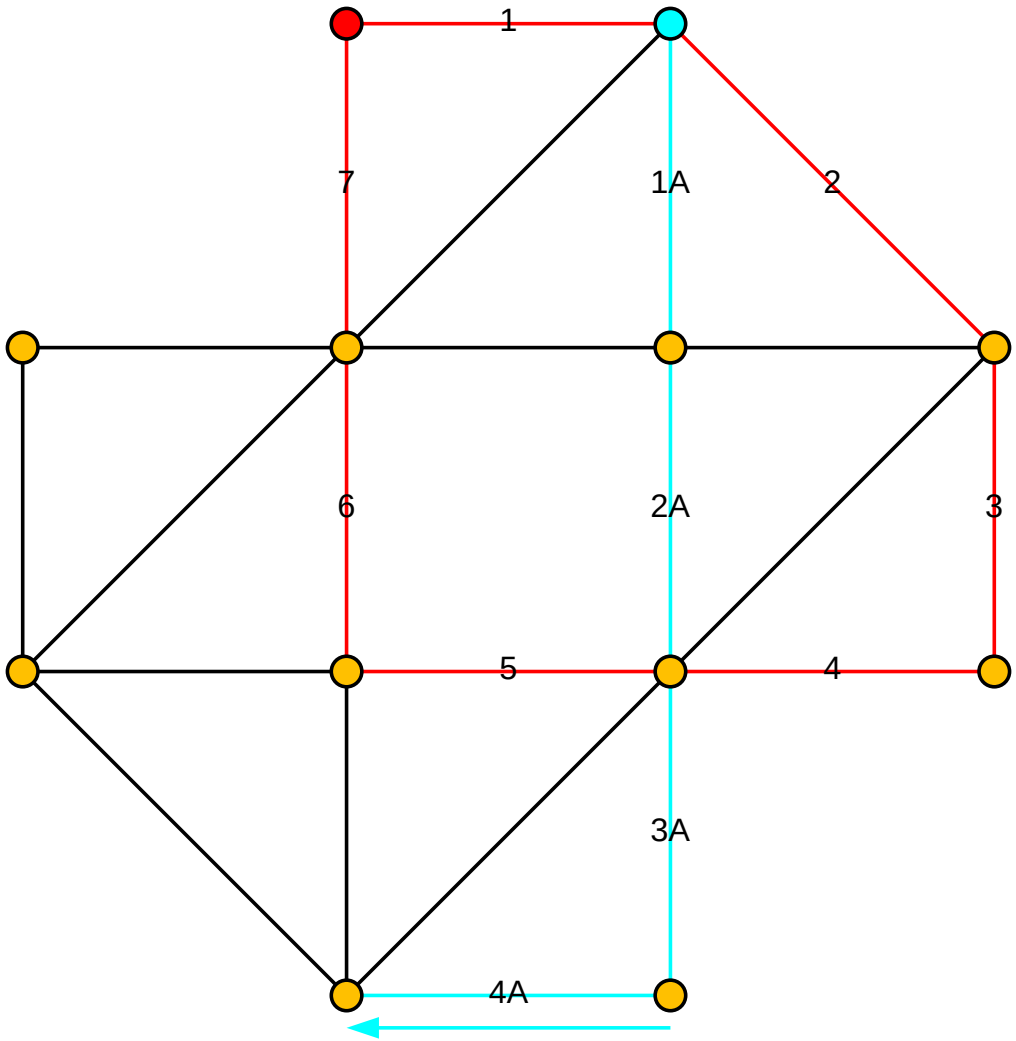

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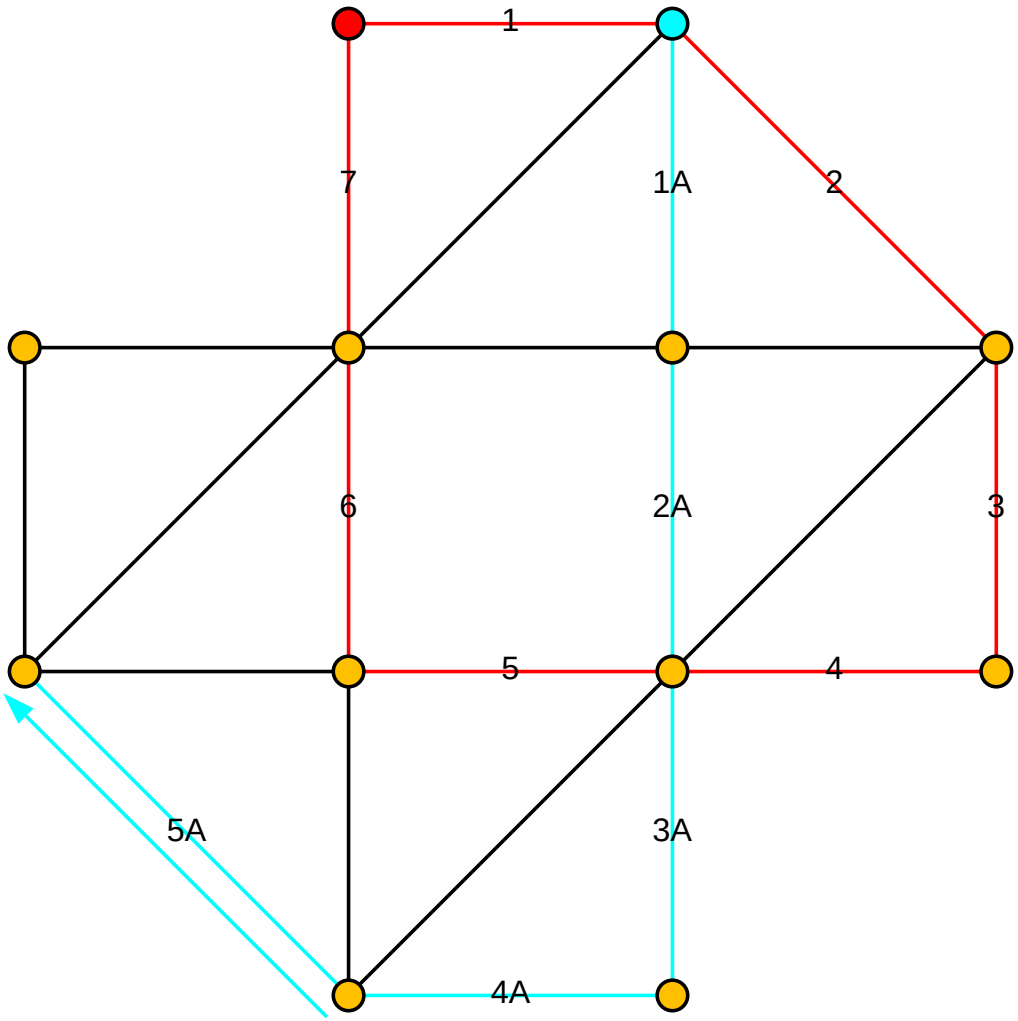

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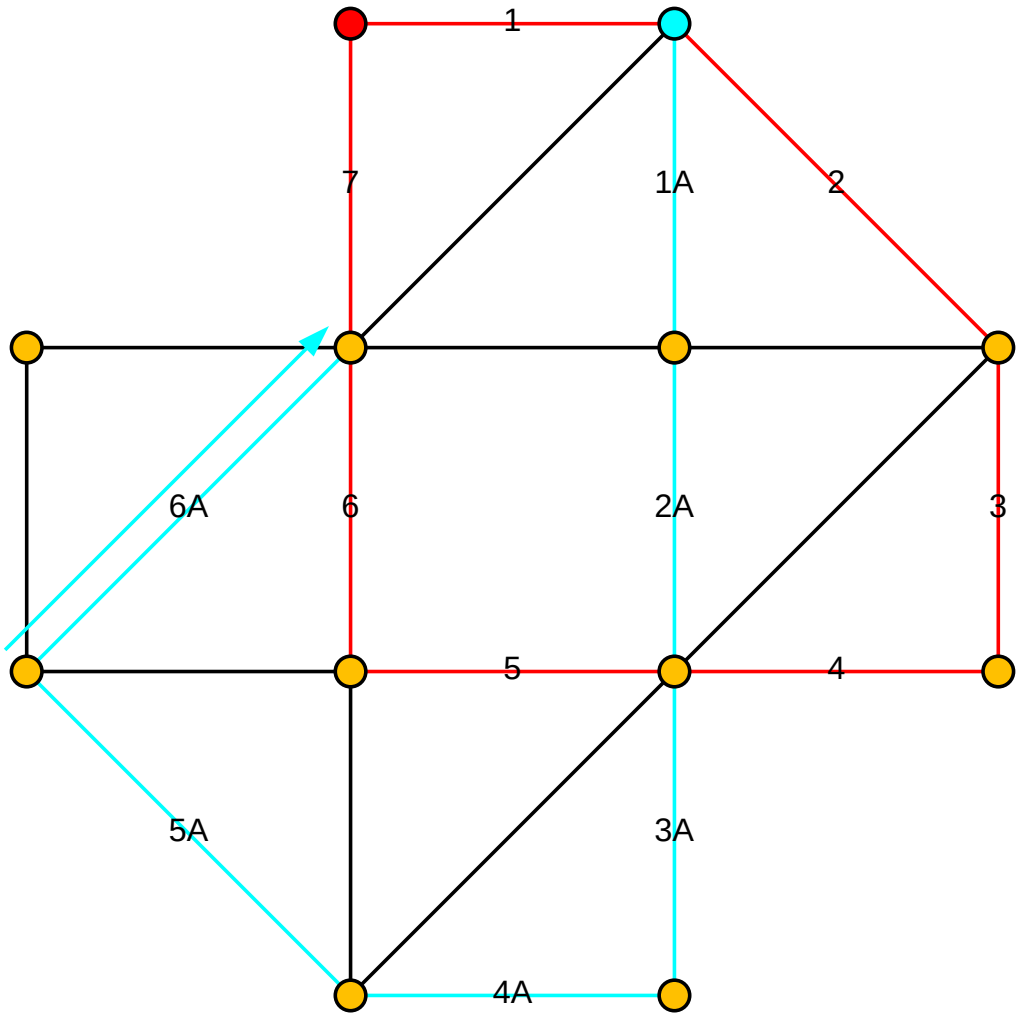

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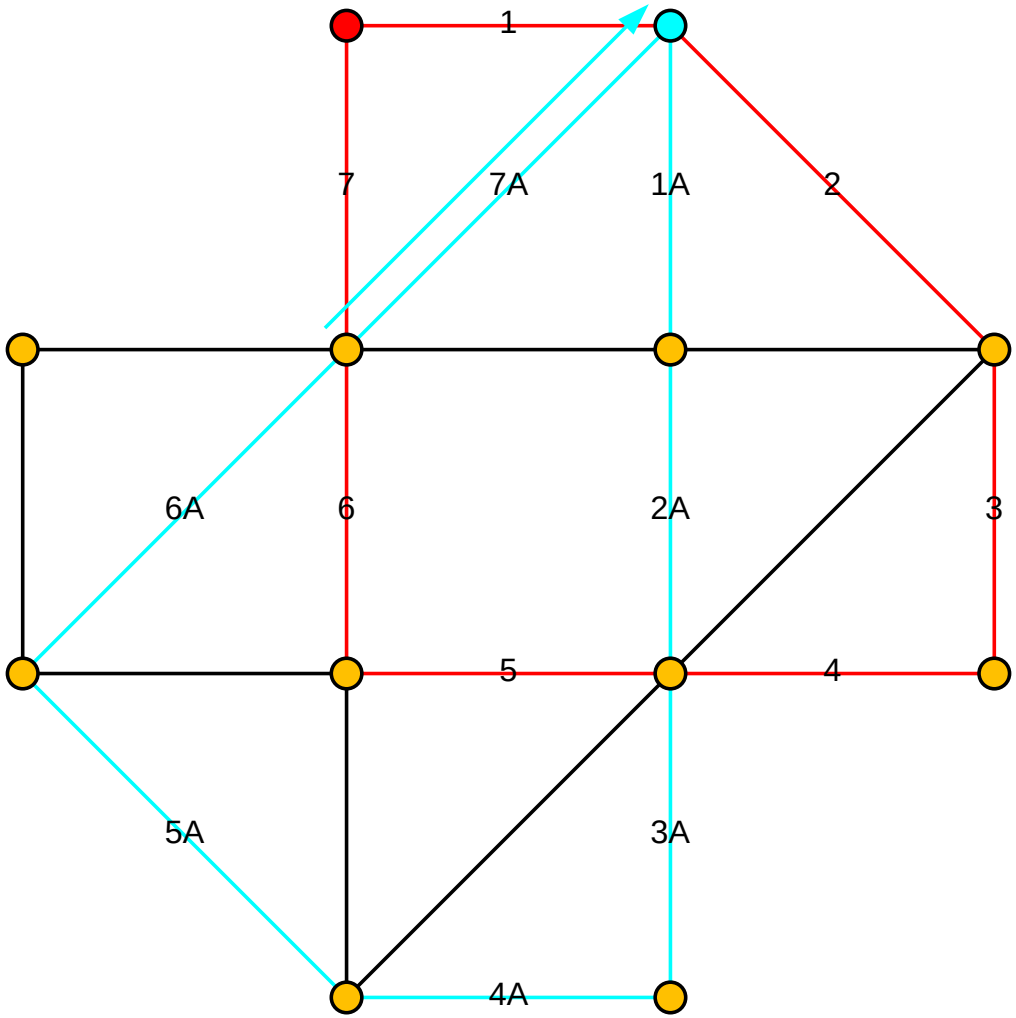

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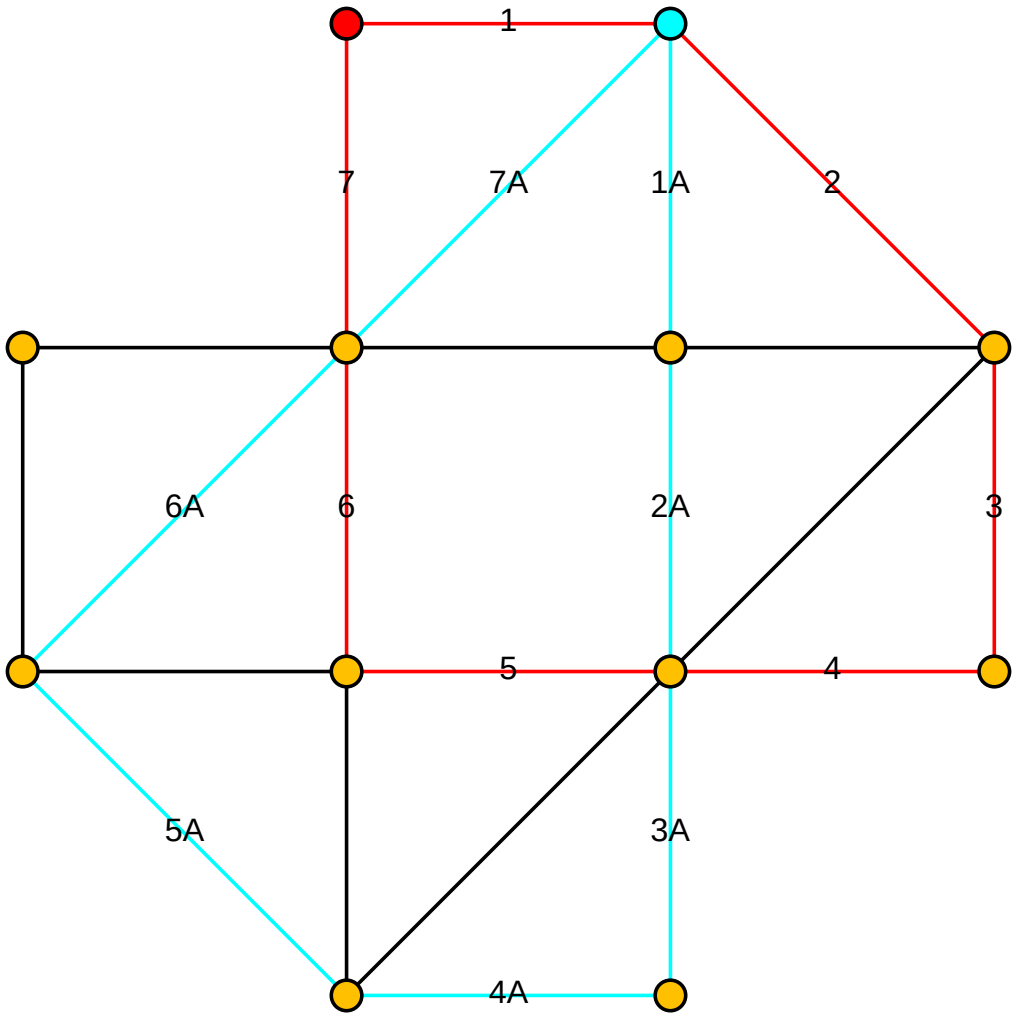

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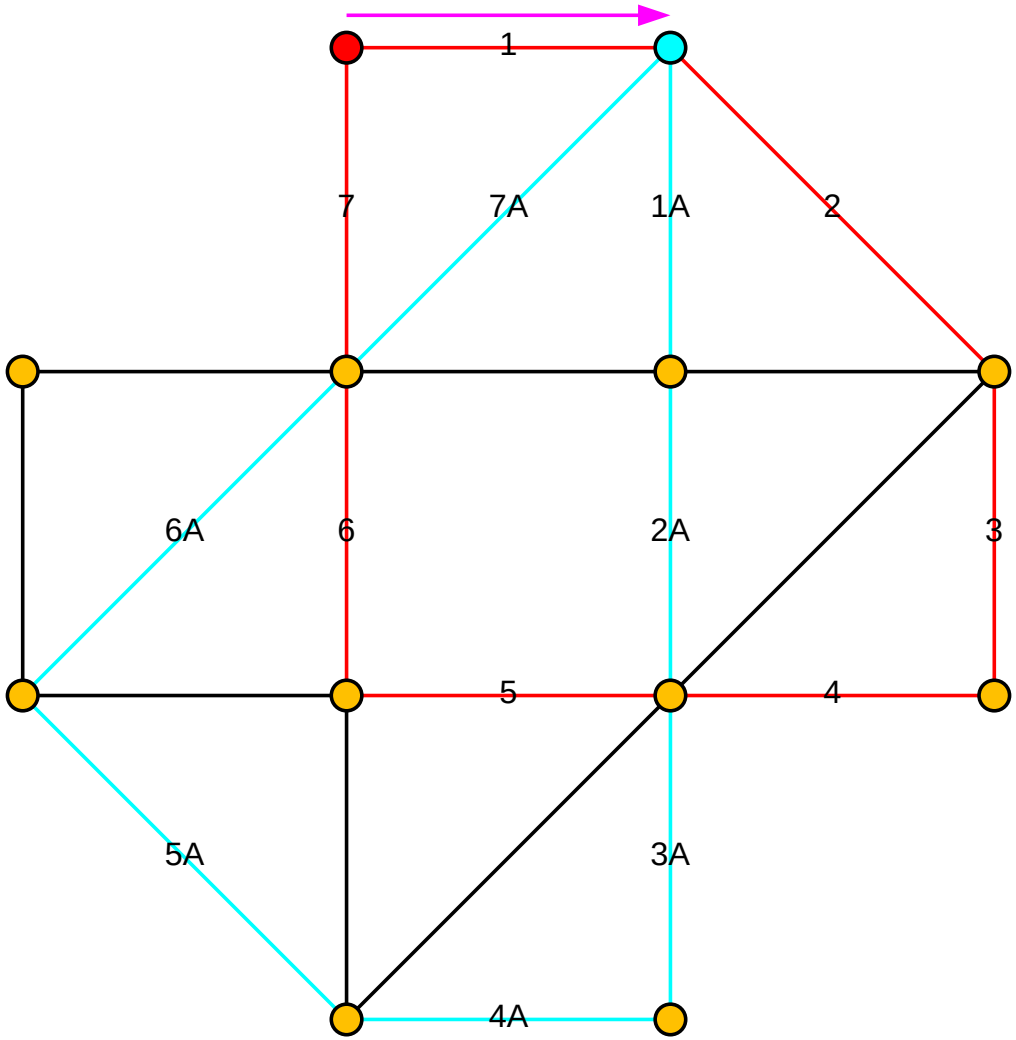

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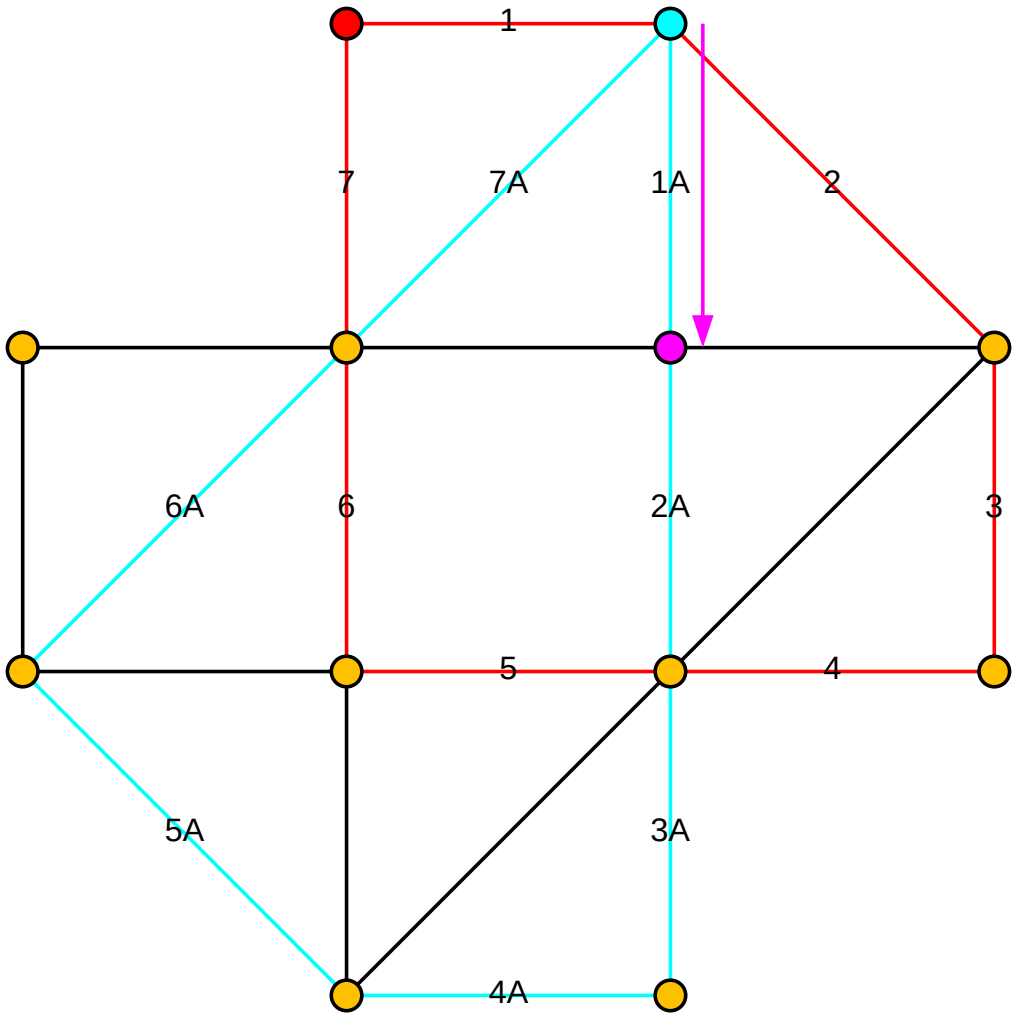

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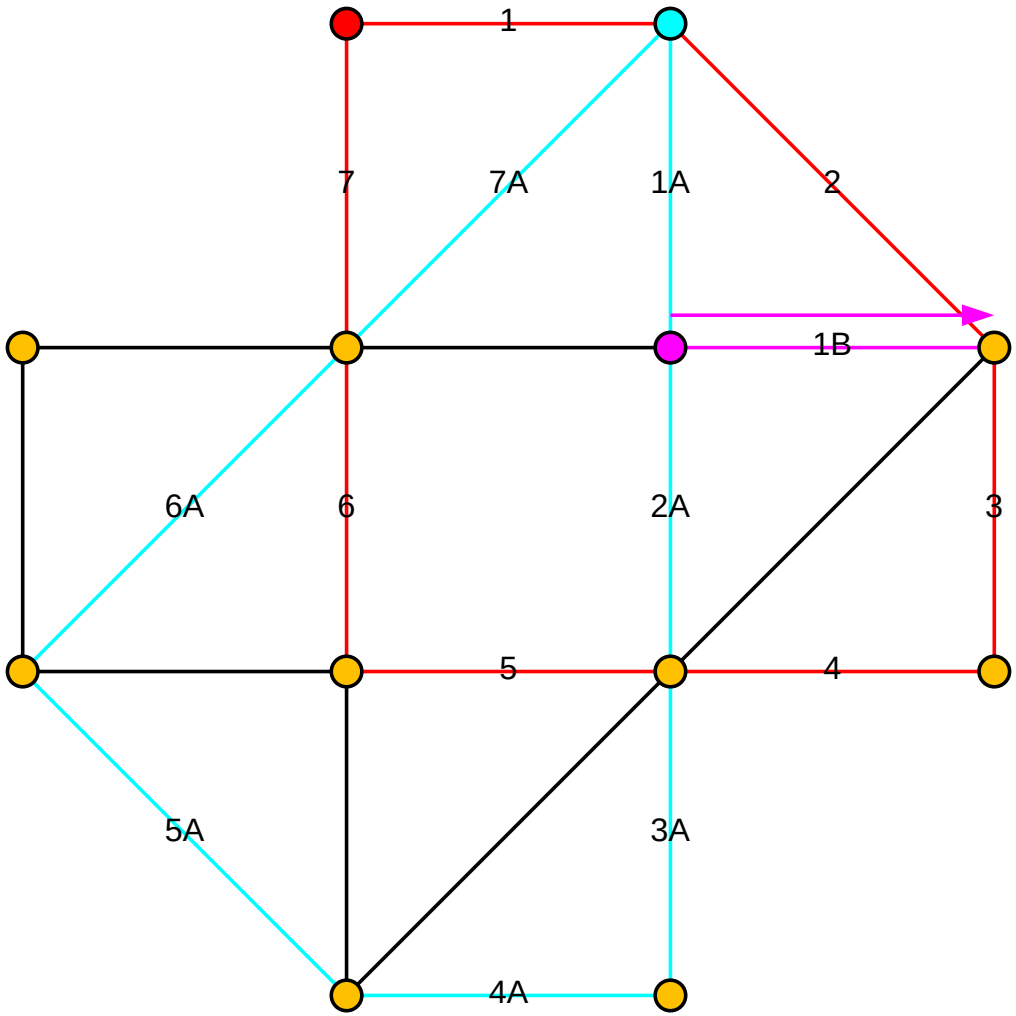

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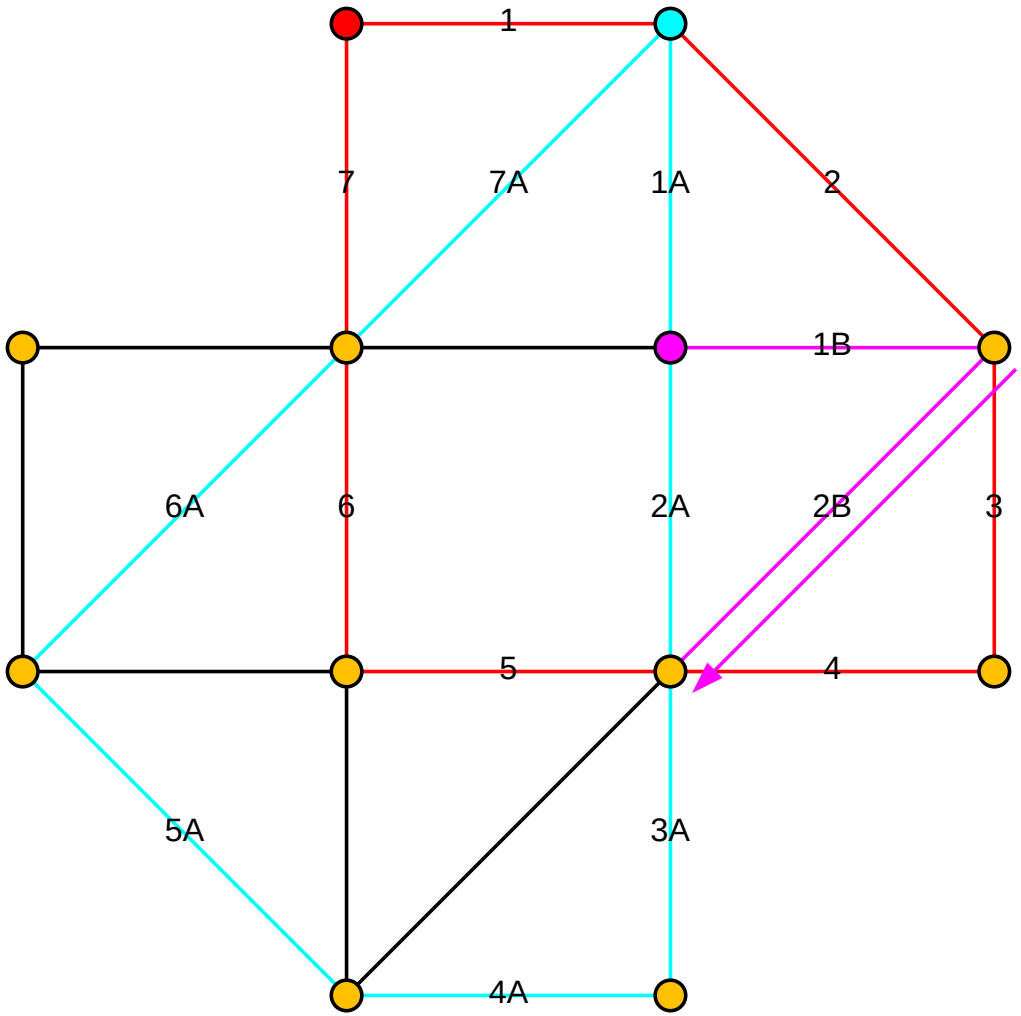
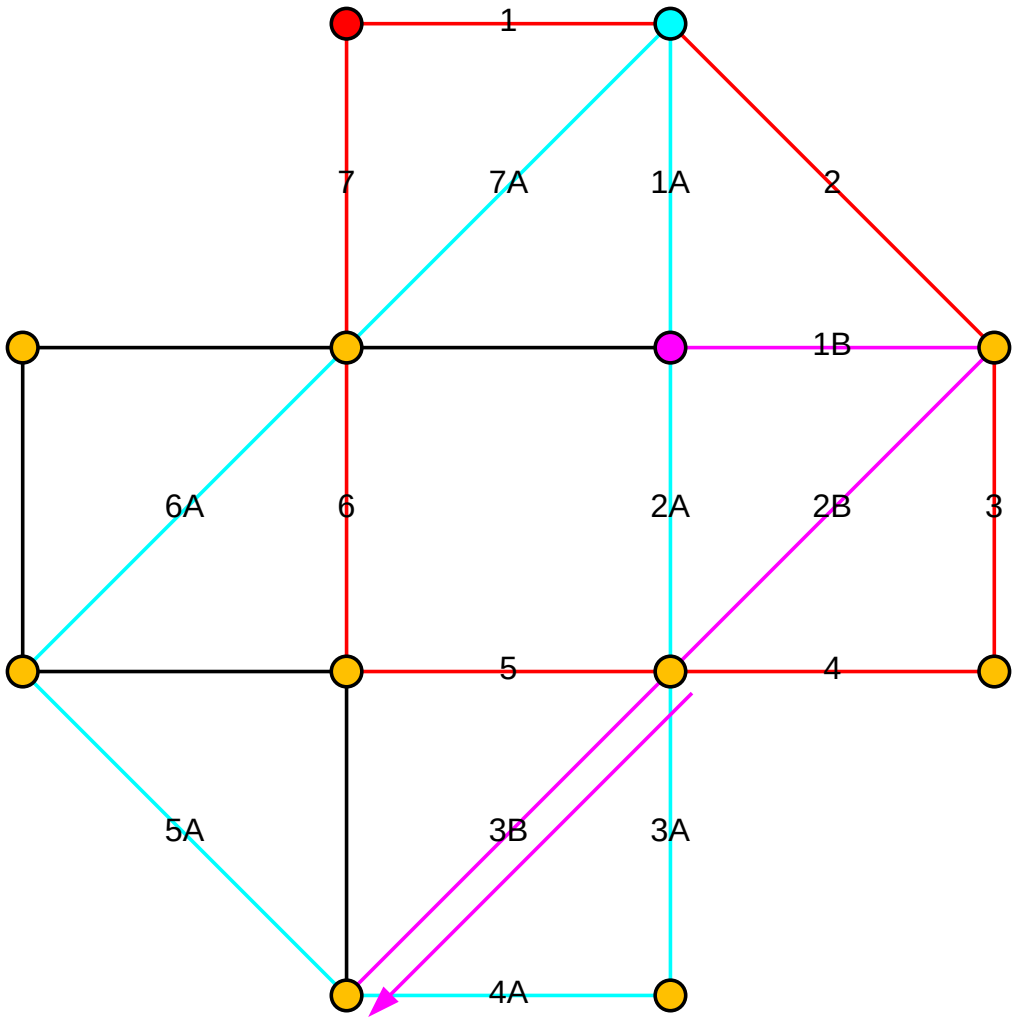
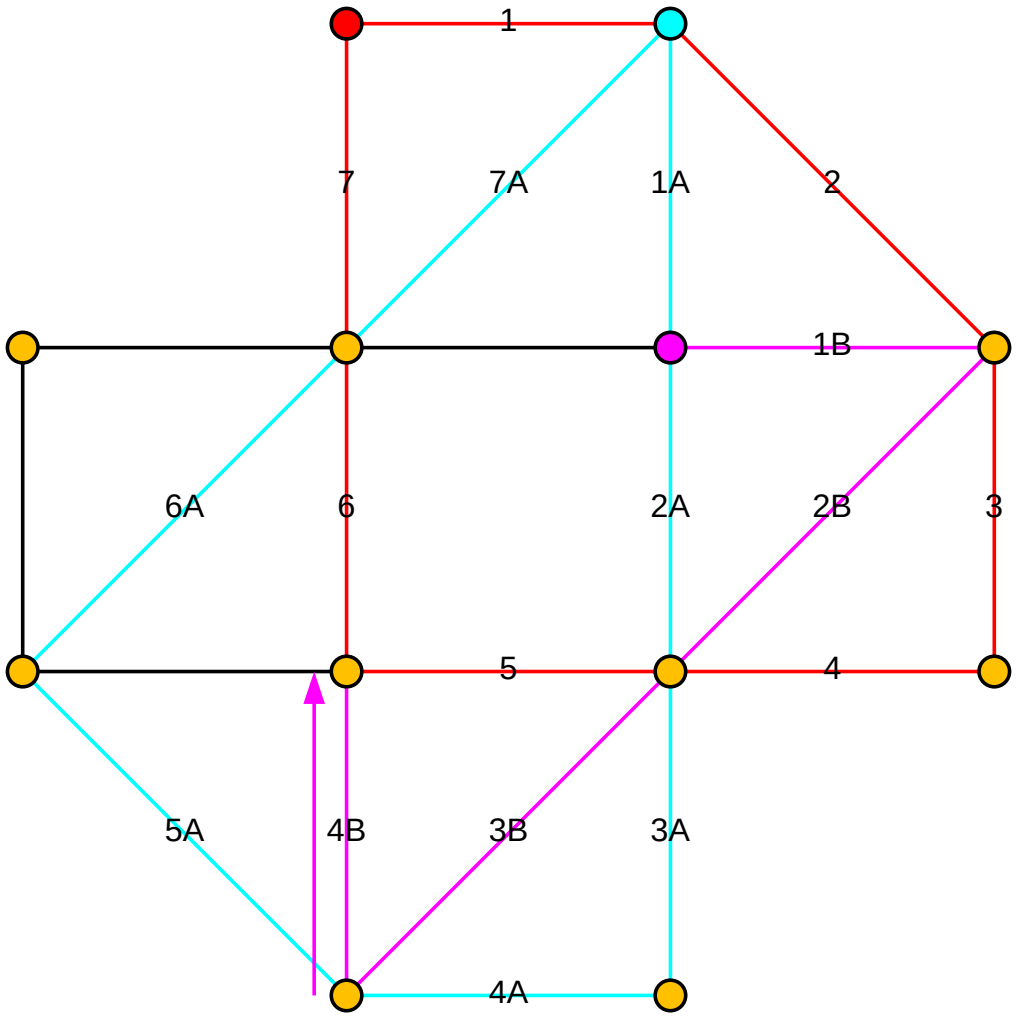


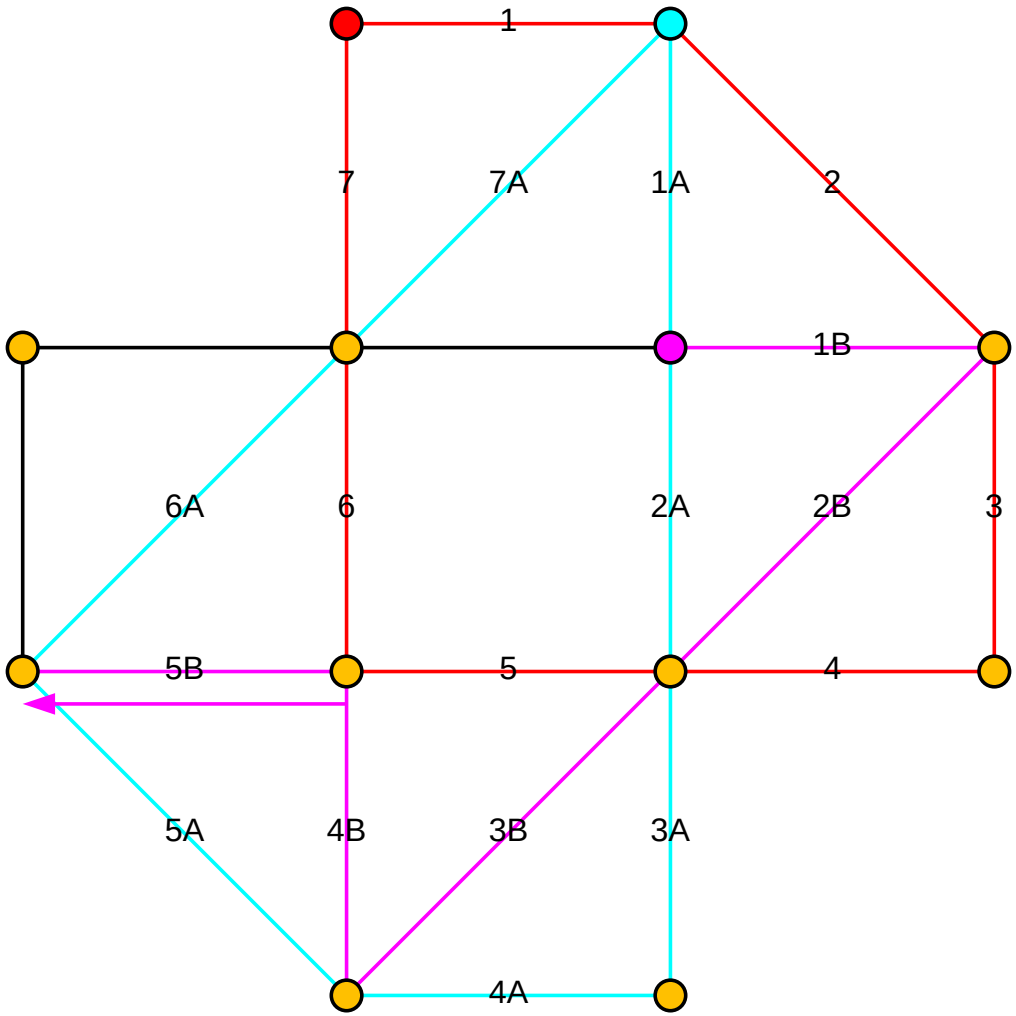
The Splicing Algorithm



The Splicing Algorithm



The Splicing Algorithm



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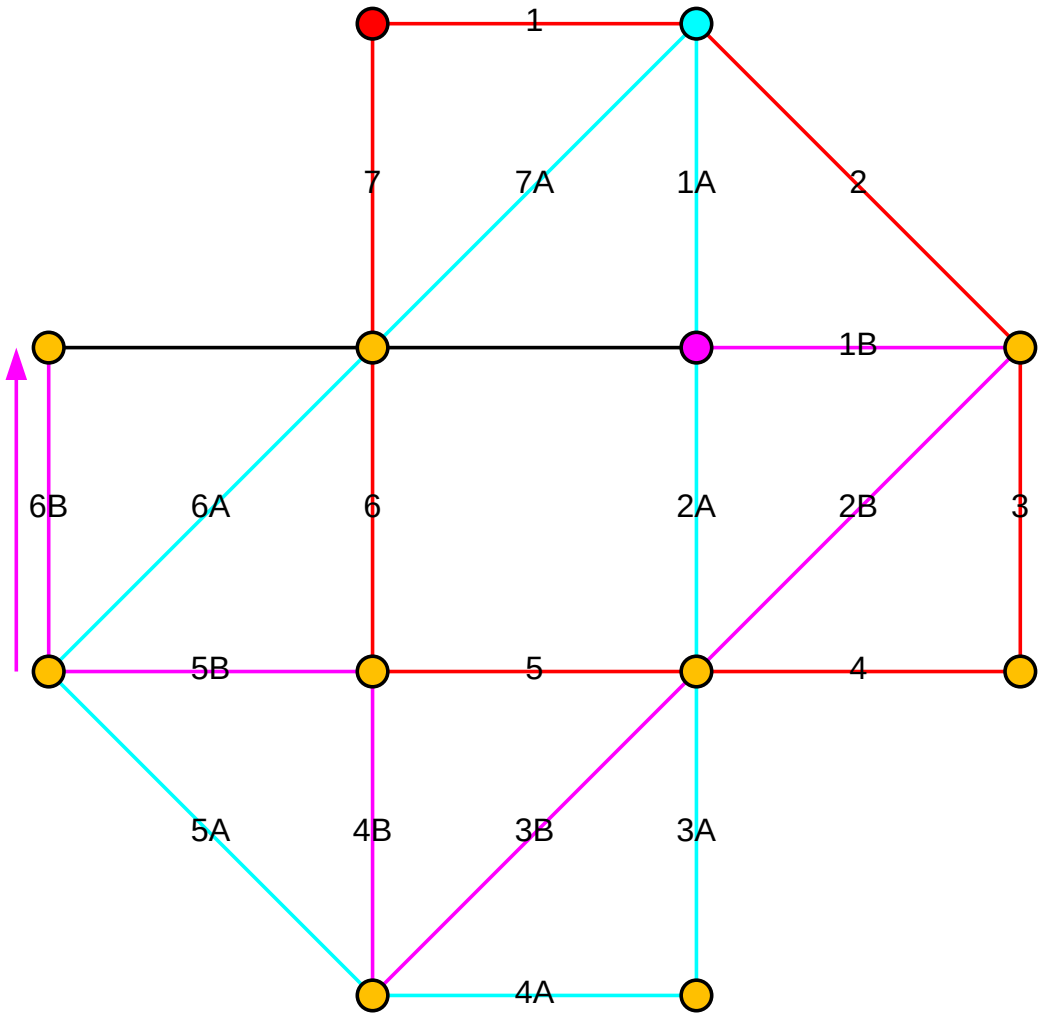

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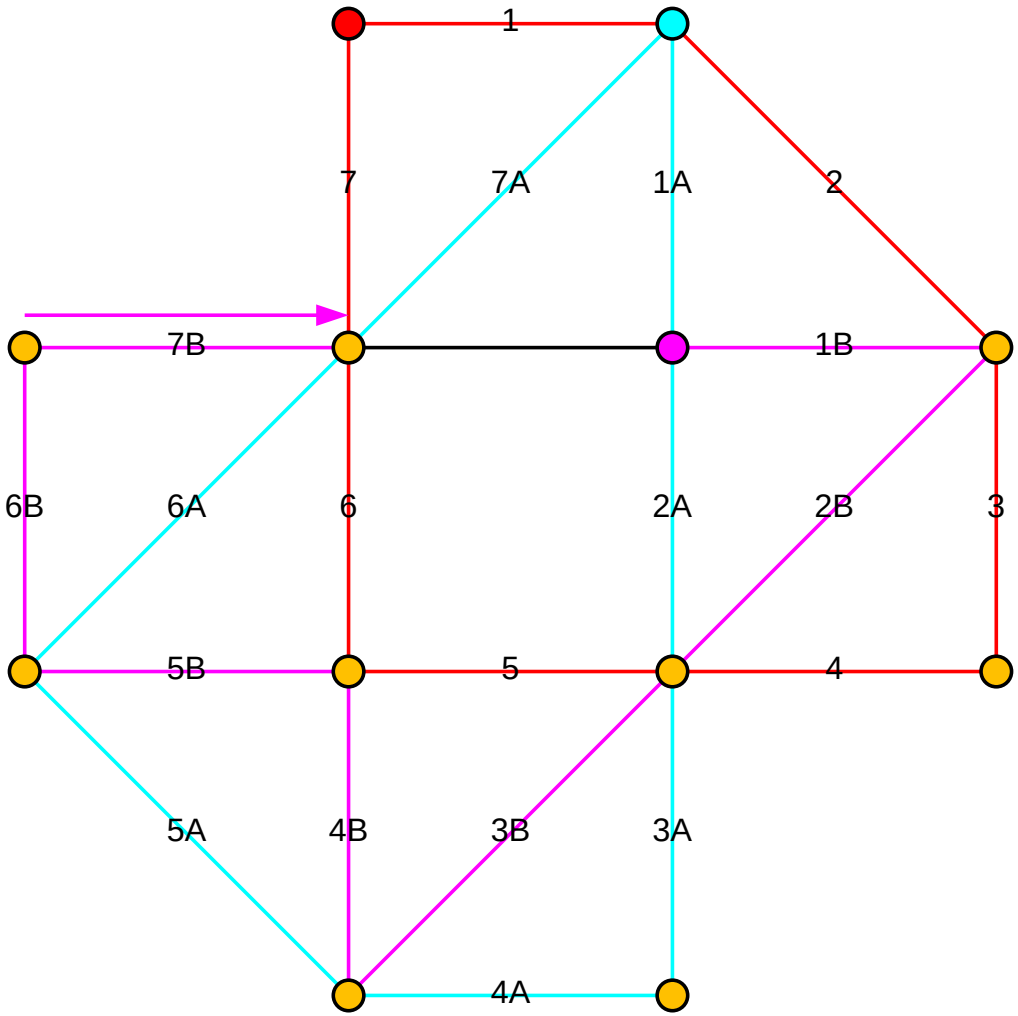

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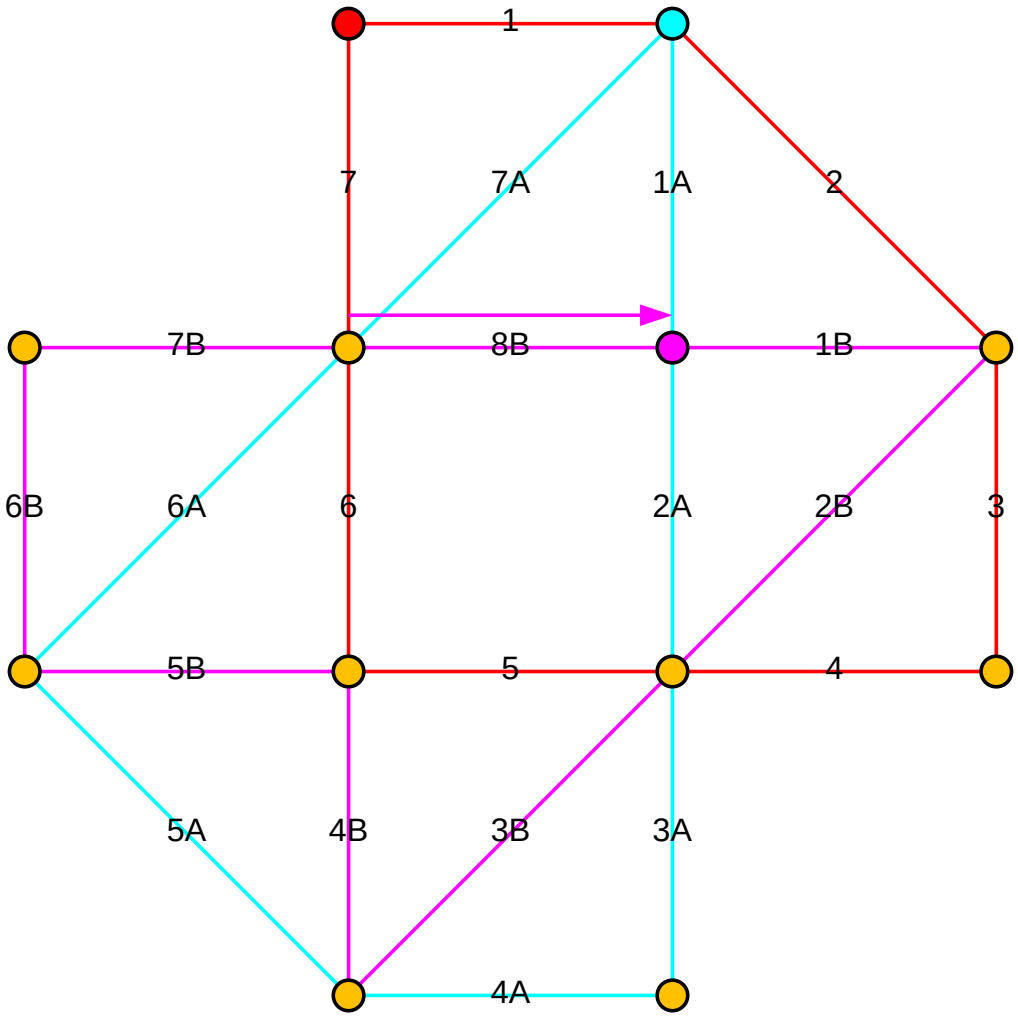
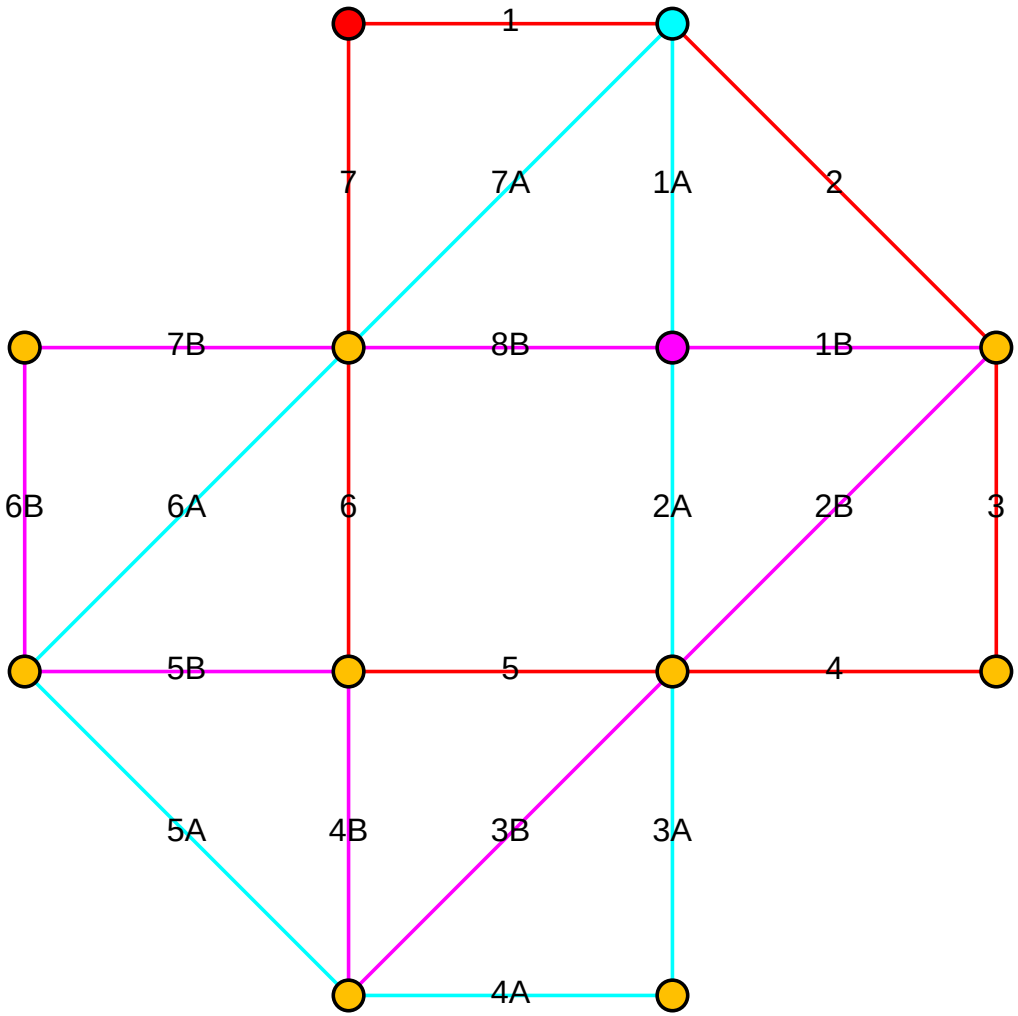


The Splicing Algorithm

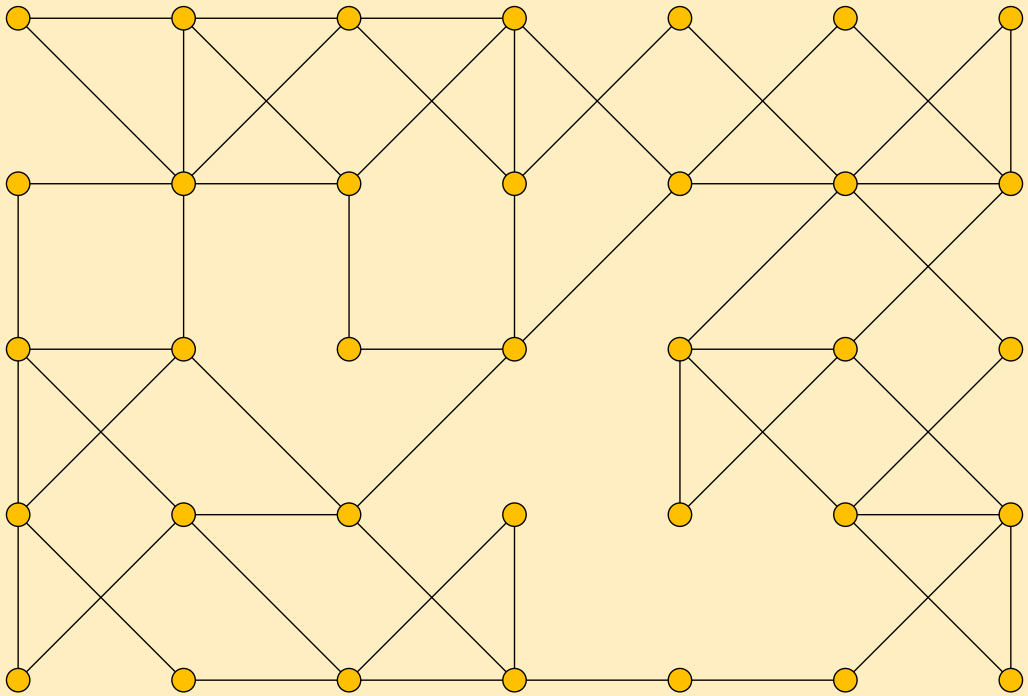


Outline

1. Definitions
2. Euler's Theorems
3. Fleury's Algorithm
4. The Splicing Algorithm
5. Example
6. The Mail Carrier Problem Solved
7. Assignment

Example

Example



- Use Fleury's algorithm and then the Splicing Algorithm to find an Euler circuit.

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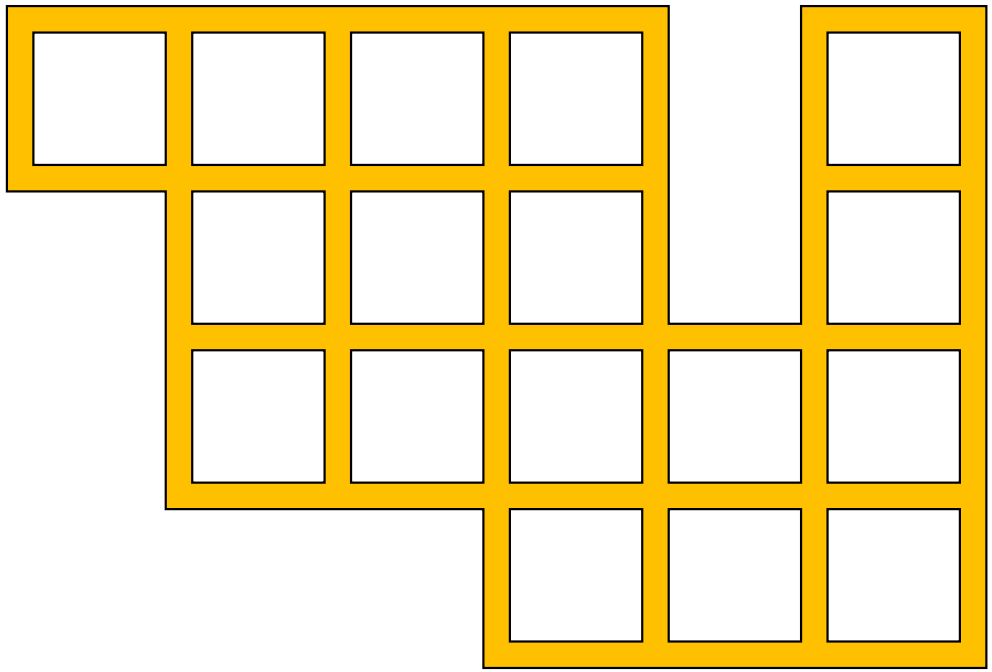
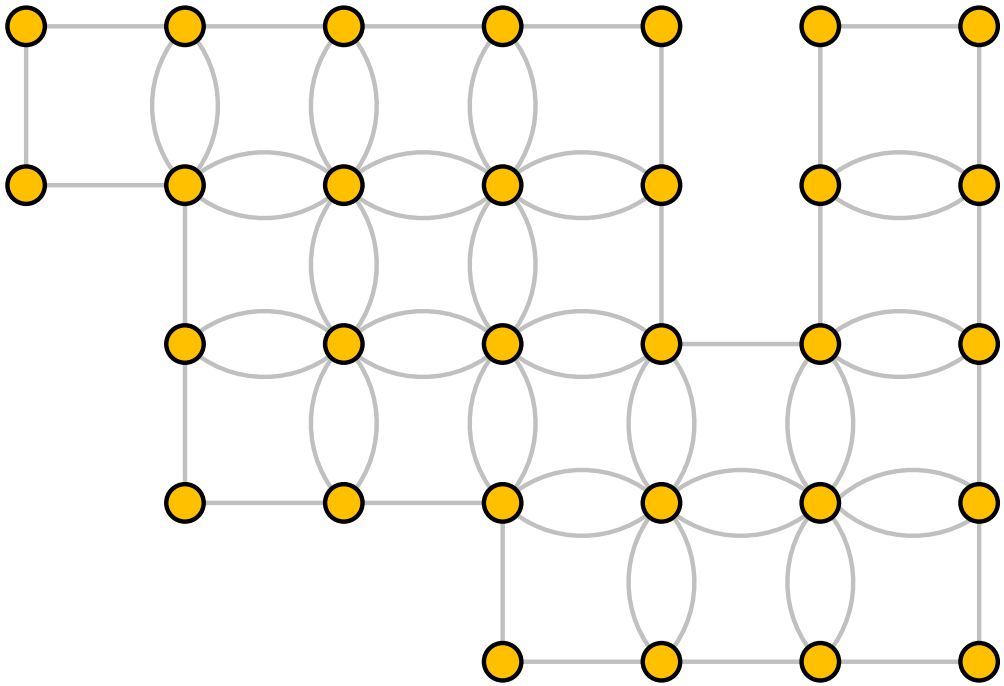
The Mail Carrier Problem Solved



- The mail carrier's graph never contains any odd vertices.

The Mail Carrier Problem Solved



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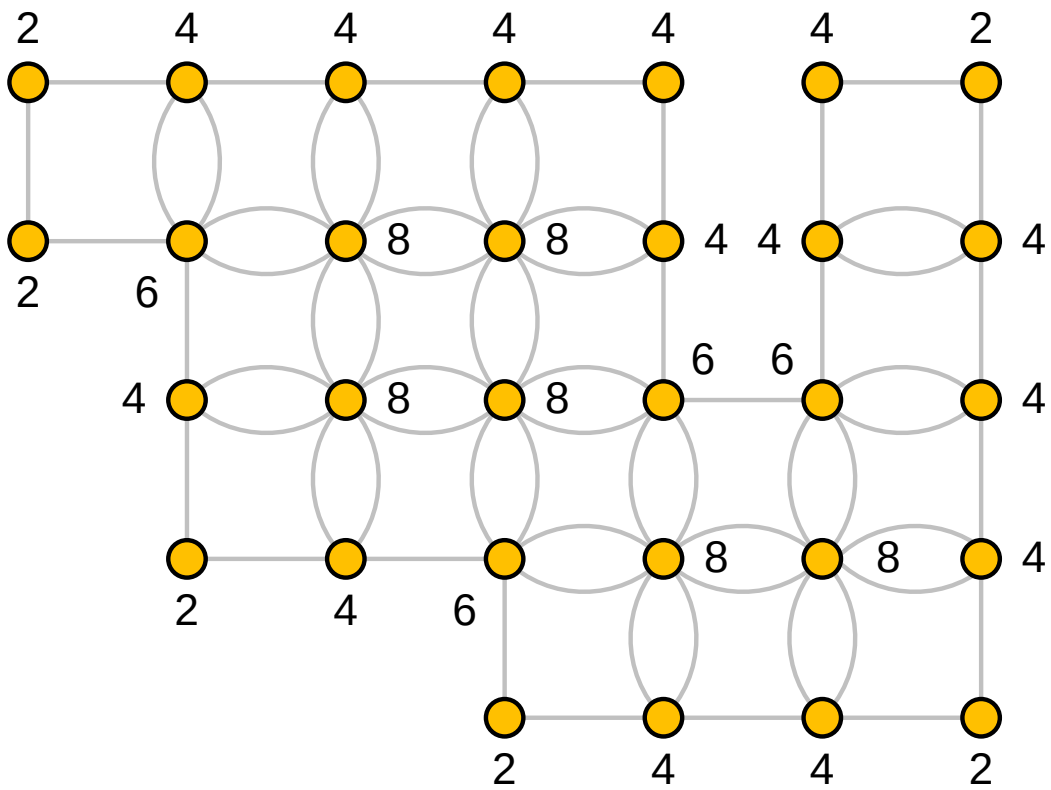
The Mail Carrier Problem Solved



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- Chapter 5: Exercises 29, 30, 31, 35, 36, 37, 38, 39.